CE323: Properties and Behavior of Materials
Fall 2003
Tuesday/Thursday, 9.30-10.45, D221 Thornton Hall

> 'Gold is for the mistress – silver for the maid –
> Copper is for the craftsman cunning at his trade.'
> 'Good!' said the Baron, sitting in his hall,
> 'But Iron – Cold Iron – is master of them all.'
>
> \- *Rudyard Kipling*

Instructor: Prof. Matthew R. Begley
begley@virginia.edu
B229E Thornton Hall
(434) 243 8728

Office Hours:
M: 10.45 – 11.45 a.m.
F: 9.00 – 11.00 a.m.

Teaching Assistant: Ms. Lilian Minja
lilian@virginia.edu
B220C Thornton Hall

Office Hours:
T: 2.00 – 4.00 p.m.
W: 2.00 – 4.00 p.m.

Course Description (catalogue): Studies the properties and behavior of engineering materials, emphasizing construction materials, including metals, concrete, wood, and composites. Considers service conditions and underlying scientific principles related to applications and performance of materials.

Course Description (Begley):

The Plot: Why do things break? Why do materials have any strength at all? Why are some solids stronger than others? Why is steel tough and glass brittle? Why does wood split? Why do we really mean by 'strength' and 'toughness' and 'brittleness'?

The Characters: Why are some roads made from concrete and others from asphalt? Why did the World Trade Centers collapse – is it reasonable to expect they should not have? Why knead bread? What did Kipling *mean* by the above poem?

The Sub-text: Are materials as strong as we ought to expect them to be? How far can we improve existing types of materials? Can we make altogether different kinds of materials [structures] which would be much stronger? If so, how, and what would they be like? How and where should we make use of them?

Course Objectives:

1. Learn the "logic of the discipline", which provides the framework to understand the impact of material properties on reliability and design.
2. Identify fundamental material deformation mechanisms that control material properties such as modulus, strength and toughness.

3. Outline strategies to control and modify microscopic deformation phenomena to improve material properties.
4. Describe key material behaviors that influence structural performance as a function of environment.

Course Text (required):

The New Science of Strong Materials, Or, Why You Don't Fall Through the Floor, J.E. Gordon, Princeton Science Library, 1976

Key References:

Materials Selection in Mechanical Design, M.F. Ashby, Butterworth-Heinemann, Oxford, England, 1996. (Strongly recommended for purchase: inexpensive paperback that is widely useful.)

Mechanical Behavior of Materials (Engineering Methods for Deformation, Fracture, and Fatigue), N.E. Dowling, Prentice Hall, New Jersey, 2nd Edition, 1999. (Recommended for purchase: standard course text with comprehensive coverage, but expensive in hardback ~ $120.)

Reinforced Concrete Design, McGraw Hill, Boston, K. Leet and D. Bernal, Massachusetts, 3rd Edition, 1995. (Not recommended for purchase (yet): standard course text that may be required later – (or a similar yet different book).)

Course Policies:

Homeworks: (8 @ 2 %)	**16%**
Quizzes: (4 @ 7.5 %)	**30%**
Midterm:	**22%**
Final:	**32%**

Homework Assignments:

The plan is to distribute homework assignments on Thursdays and have them turned in by the Friday of the following week. Specific details will be included with each assignment. Current plans are for eight homework assignments of 3-4 problems (an unspecified fraction will be graded).

Homework is not pledged but should never be copied. Students may consult each other and share ideas in the ***early*** stages of homework solutions, but each student must complete his/her own homework independently. Homework that is identical in all ***significant*** respects is unacceptable.

A due date will be given when homework is assigned, and will typically be the Friday of the week following the homework's distribution . Homework turned in after the due date will be accepted only if arrangements are made prior to the original due date; this will only be possible under extremely extenuating circumstances.

Students should regard homework problems as professional presentations. Homework should be neatly organized, with the answers to problems summarized at the appropriate locations. Sufficient detail must be included to permit satisfactory evaluation of student performance, but excessive detail should be avoided.

Tests:

1. All tests and examinations will be administered under the University of Virginia honor system. Students will be assumed to be familiar with the honor system, and will be bound by it. The honor system is a very important attribute of the University of Virginia, but only works if the concept of honor is taken seriously by all involved.

2. The detailed formats of tests will be announced prior to their administration, and specific limits within which the student is permitted to work will be announced. Time permitting, review sessions will be held in the evening, prior to each exam.

Miscellaneous:

Please turn off cell phones. Penalties will be decided on the spur of the moment and may be egregiously disproportionate to the offense.

Use e-mail to make an appointment with me if you cannot attend office hours.

CE363: Materials Laboratory
Fall 2003
Tuesday: 2.00-3.30 p.m, 3.30-5.00 p.m., Thursday: 2.00-3.30 p.m.

Instructor:
Prof. Matthew R. Begley
begley@virginia.edu
B229E Thornton Hall
(434) 243 8728

Office Hours:
T: 10.45 – 11.45 a.m.
W: 9.00 – 11.00 a.m.

Course Description: Laboratory study of the macroscopic mechanical, thermal, and time-dependent properties and behaviors of typical civil engineering construction materials (metals, concrete, wood, plastics). Students observe, plan and conduct experiments, and prepare written reports.

Course Objectives:

1. Learn "the logic of the discipline", which provides the framework to understand the connections between material microstructure and mechanical properties.
2. Identify fundamental material deformation mechanisms that control material properties such as modulus, strength and toughness.

Course Text:

None. Laboratory handouts/reading will be distributed before each lab.

Key References:

Materials Selection in Mechanical Design, M.F. Ashby, Butterworth-Heinemann, Oxford, England, 1996. (Strongly recommended for purchase: inexpensive paperback that is widely useful.)

Mechanical Behavior of Materials (Engineering Methods for Deformation, Fracture, and Fatigue), N.E. Dowling, Prentice Hall, New Jersey, 2nd Edition, 1999. (Recommended for purchase: standard course text with comprehensive coverage, but expensive in hardback ~ $120.)

Course Policies:

Laboratory Reports/Exercises: ~~4~~ 6 @ ~~20~~ 15%…………….~~80~~% 90
Final Exam: ………~~20~~% 10

Laboratory Reports:

Specific details regarding the content and expectations of each laboratory report will be included with each lab handout. Current plans are for six assignments: four laboratory reports on traditional experiments, and two "thought experiments" which will be conceptual exercises regarding the design of experiments.

Students may consult each other and share ideas in the ***early*** stages of laboratory report development, but each student must complete his/her own report independently. Reports that are identical in all ***significant*** respects are unacceptable.

Students who miss the laboratory section during which experiments are run and data is collected will not be considered to have conducted the experiments. This implies that experimental data may not be included in their reports, and will be considered illegally obtained, unless arrangements are made with the instructor *prior* to the lab. It is difficult to imagine an effective lab report (i.e. one that receives a grade that differs from temperature reading in Celcius) – so please do not miss lab. Reports turned in after the due date will be accepted only if arrangements are made prior to the original due date; this will only be possible under extremely extenuating circumstances.

Students should regard reports as professional presentations. They should be well-written, ***typed*** and with ***computer-generated graphics/plots***. They should be neatly organized, with the key conclusions summarized at the appropriate locations. Sufficient detail must be included to permit satisfactory evaluation of student performance, but excessive detail should be avoided.

Final Exam:

1. The final will be administered under the University of Virginia honor system. Students will be assumed to be familiar with the honor system, and will be bound by it. The honor system is a very important attribute of the University of Virginia, but only works if the concept of honor is taken seriously by all involved.

2. The laboratory final will be a one hour exam during the final week of class.

Miscellaneous:

Please turn off cell phones. Penalties will be decided on the spur of the moment and may be egregiously disproportionate.

Use e-mail to make an appointment with me if you cannot attend office hours.

CE323 Properties and Behavior of Materials Fall 2003
Remaining Schedule

Quizes/Homeworks

Tuesday, 11/18	HW #5 DUE
Thursday, 11/20	*QUIZ #3: Fracture Toughness (open notes/book)*
Tuesday, 11/25	HW #6 DUE
Tuesday, 12/2	*QUIZ #4: Strengthening*
Thursday, 12/5	HW #7: DUE

Laboratory Reports:

Experiment #5: Tuesday, 12/2, and Thursday, 12/5

Friday, 12/5	Laboratory Reports 3&5: Charpy and Wood Tests (Early birds: turn in Lab #3 on 11/25)
Friday, 12/12	Laboratory Report #6: Concrete Testing Extra-credit: completed Experiment #4: Cheese fracture tests

Final Exam: Friday, December 12th, 2-5 p.m.
(CLOSED BOOK/NOTES)

CE 323: QUIZ #1

Catherine Hovell
September 16, 2003

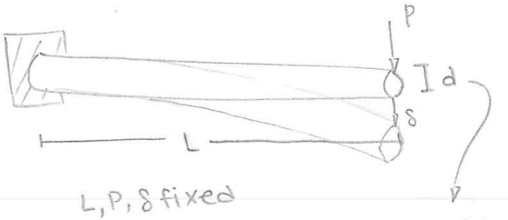
1. Fixed: load
allowable deflection
length
(width)
(doesn't exist if beam is circular and "thickness" can change)

variable: cost / volume
strength / density

$\rho = \frac{m}{V}$

needed: lightest beam possible
(that can handle fixed requirements)

An optimal material would be strong while having a low density. This allows for the beam itself to be lightweight and therefore minimize deflection due to its own weight. To compare materials with similar strength-density relationships, the cost / unit volume should be considered. For each material being considered, some mass is necessary to withstand the load without exceeding the allowable deflection. From this mass, volume and cost estimates can be made.



can change; related to mass, volume and strength

+6

2. ◦ strength – Pa, psi – σ : strength of a material relates to its ability to withstand flaws within a section. (strong vs. weak) +1

◦ stiffness – no units – E : stiffness relates to the ability of a material to deform elastically (a stiff material will break, a flexible one can be strained to high percentages before failure). +1

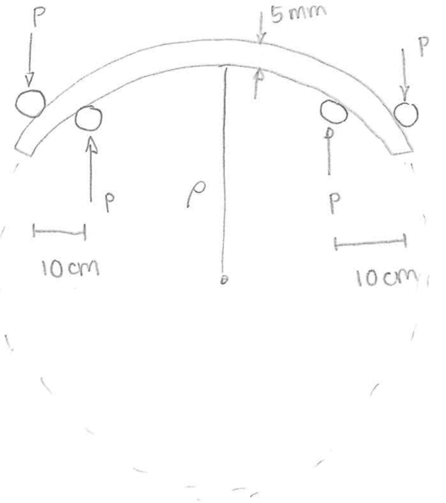
◦ toughness – : toughness relates to the ductility of a material and whether or not it can stretch without failing. copper is tough, or ductile – it can be stretched into a wire, whereas concrete is brittle and would fail under tensile stress.

+2

6
5
8
$\boxed{19}$

Catherine Hovell

3.



E = 4 GPa

b = 2.5 cm

$\varepsilon = \frac{y}{\rho} = \frac{\sigma}{E}$

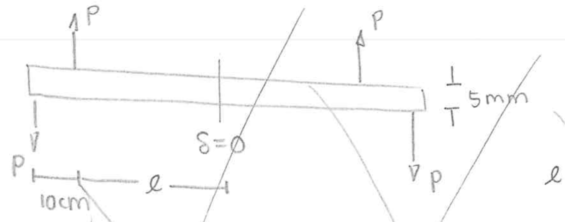
$\sigma = E\frac{y}{\rho}$ $y = \frac{1}{2}d = 2.5$ mm

$\rho = 6d = 30$ mm

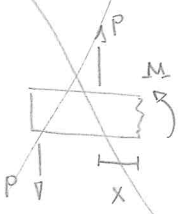
$\sigma = E\frac{\frac{1}{2}\cancel{d}}{6\cancel{d}} = \frac{E}{12}$ +7

$\sigma = \frac{4\times10^{9}\ \text{Pa}}{12} = 0.33$ GPa

$\boxed{\text{a. } \sigma \approx 330 \text{ MPa}}$



~~ℓ ≈ 40 cm~~



~~V = 0~~

~~$M = Px - P(x+10\text{cm}) = -P(10\text{cm})$~~

~~$EIv'' = M$~~

~~$EIv' = -10Px + C_1$ $v' = 0$ at $x = \ell$, $C_1 = 10P\ell$~~

~~$EIv = -5Px^2 + C_1x + C_2$ $v = 0$ at $x = \ell$, $C_2 = -5P\ell^2$~~

where am I going....

$\sigma = \frac{P}{A}$ $A = (5\text{mm})(2.5\text{cm}) = 1.25\times10^{-4}\ \text{m}^2$

$P = \sigma A$ $P = (330\times10^{6}\ \text{Pa})(1.25\times10^{-4}\ \text{m}^2)$

$P = 41000$ N

$P = 41$ kN

$\boxed{\text{b. } P \approx 40 \text{ kN, or 40x my colleague's guess}}$

+1

CE323 Properties and Behaviors of Materials **Quiz 1**
Fall 2003 **September 16th, 2003**

1. You need to select a material for a ~~component~~ circularshaft subjected to bending, to be used in a Formula One racing car. Derive the combination of material properties that should be maximized to make the lightest ~~spring~~ beam possible for fixed values of load, allowable deflection, length and width. (I.e. the thickness of the beam (hint hint) can be adjusted once the material is selected.)

2. Explain the difference between strength, stiffness and toughness. Indicate the engineering variables that correspond to each property.

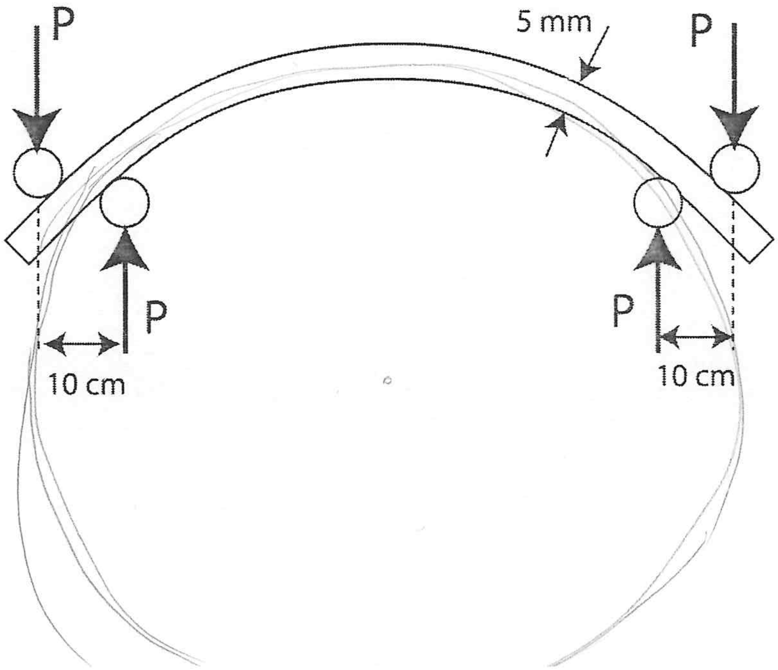
3. A thin strip of Plexiglass [poly (methyl methacrylate) – PMMA] is tested in four-point bending to determine its strength. The thickness of the strip is 5 mm, its width is 2.5 cm and the distance between the inside and outside loading pins is 10 cm. The modulus of PMMA is ~4 GPa. Just before failure, the computer acquisition system crashed (undoubtedly a Microsoft operating system), so that loads and displacements were not recorded.

   a. The boss is screaming for an estimate of the material's failure stress, and there's no time to re-do the test. Luckily, you were clever enough to video tape the test, and the shape of the strip just before failure is shown below. Provide an estimate for the strength of the PMMA.

   b. Your colleague (a VA Tech grad) thinks the load cell read P = 1 kN at failure; impress your boss as to the quality of your education by estimating the true failure load, ignoring for the sake of approximation the effect of large geometry changes.

Extra Credit: In 1676, Robert Hooke informed the world he intended to publish a theory on the deformation of materials: what did he call his theory?

on my honor as a student,
I have neither given nor
received aid on this quiz.
Catherine S Ham



QUIZ #1: Attempt 2

Catherine Hovell
September 23, 2003

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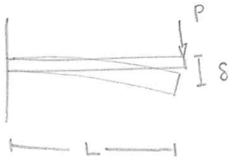
1. fixed: load
deflection
length

k_{des} ≡ design stiffness $= \frac{P_{max}}{\delta_{allow}}$

$\sigma = \frac{P}{A}$ $\varepsilon = \frac{\delta}{L_o}$ $\sigma = E\varepsilon$

$\frac{P}{A} = E\frac{\delta}{L_o}$ $\delta = \frac{PL}{EA}$



$\delta = \frac{PL}{EA}$

E, A are only variables

$\delta EA = PL$ or $kL = E\frac{m}{\rho L}$

$kL^2 = m\frac{E}{\rho}$ $m = kL^2\frac{\rho}{E}$

$k = \frac{P}{\delta}$ $EA = kL$ $A = \pi/4\, d^2$

$k\frac{m}{\rho} = E(\pi/4\, d^2)^2$ $m = \rho V = \rho LA$

so, minimize $\frac{\rho}{\sqrt{E}}$ to get maximum stiffness with minimum weight.

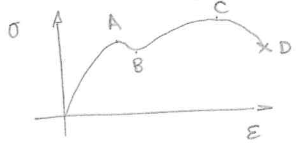
confusing +6

ultimately, end up with an equation where $m \propto \frac{\rho}{E^{1/2}}$ (something's already wrong with my eq., as I have $m \propto E\rho$.)

2. ◦ strength (strong vs. weak) – psi, Pa – maximum load carried by a given area, or maximum stress before permanent deformation / failure.

(flaw tolerance)

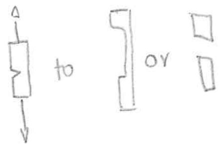
+10



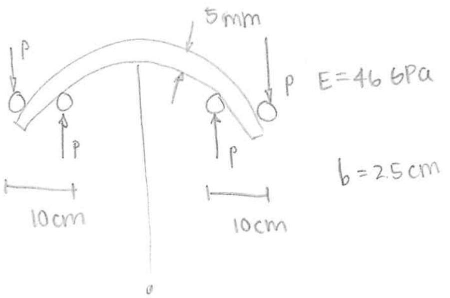
A: last point of recoverable deformation σ_y
C: ultimate (tensile) strength, σ_{uts}
D: fracture stress, strength, σ_{frac}

◦ stiffness (stiff vs. flexible) – lb/in, N/m – relates forces and displacements (stresses and strains)
modulus – due to bonding, structure

◦ toughness (tough vs. brittle) – symbol: k – related to ductility, deformation before failure.
The ability of a material to withstand flaws or defects; related to the energy required to make new surfaces, or advance a crack.



3.



E = 46 GPa

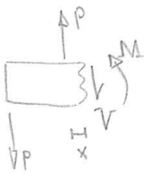
b = 2.5 cm

$\varepsilon = y/\rho = \frac{\sigma}{E}$ $\sigma = E\frac{y}{\rho}$ $y = \frac{1}{2}d = 2.5\,mm$ $\rho = 15d = 75\,mm$

$\sigma = \frac{1/2\, d}{15d} \cdot E = \frac{E}{30}$

$\sigma = \frac{4 \times 10^9\, Pa}{30} = 133.3\, MPa$

$\boxed{a.\ \sigma \approx 130\, MPa}$



V = 0
M = Px – P(x + 10cm)
M = 10P

$\sigma = \frac{Mc}{I} = \frac{10Pc}{I} = 130\, MPa$

$\frac{(130\, MPa)(1/12 \cdot (2.5cm)(0.5cm)^3)}{(10cm)(0.25cm)} = P$

P = 135.4 N

+10

$\boxed{\text{b. } P \approx 135\,N, \text{ an order of magnitude smaller than my colleague's guess}}$

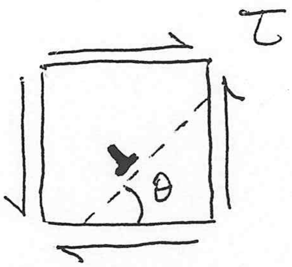
Catherine Hovell

19

CE323 Properties and Behaviors of Materials **Quiz 2**
Fall 2003 **October 9th**

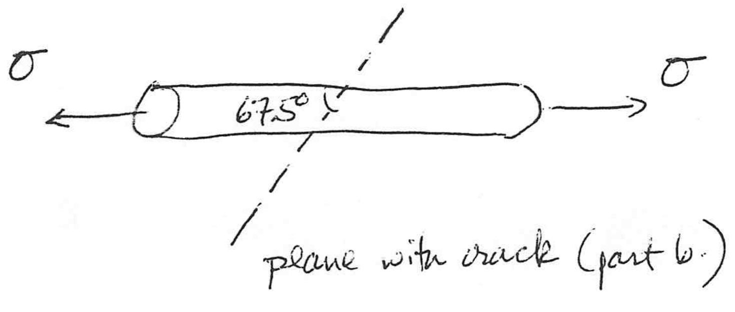
1. A block of metal is loaded in pure shear, and has a single dislocation, whose orientation is defined by angle θ from the horizontal. Mohr's circle

   a.) Explain the difference between the strength of the test *specimen* and that of the material itself.
   b.) Determine and sketch the relationship between the applied shear stress, the critical stress needed to cause dislocation motion, and the orientation angle of the slip plane.
   c.) Sketch the deformed shape of the block just after yield if there are multiple dislocations with a broad range of orientation.



2. A piece of chalk is 6 mm diameter. Assume the elastic modulus is E= 70 GPa, and the fracture toughness is $G_c = 2$ J/m^2.

   a.) If there is a flaw in the chalk and the chalk fails when subjected to ~~22~~ 220 N of tensile axial force, what is the minimum possible flaw size?
   b.) Suppose I have a second magical piece of chalk with the same properties and only one flaw with a total length ~ 160 μm (located in the center of piece), *and* suppose that the flaw is oriented 67.5° from the loading axis. What is the tensile force that I must apply to break the chalk?
   c.) Comment on the presence of energy absorption mechanisms during fracture in this material.

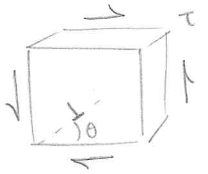


on my honor as a student, I have neither given nor received aid on this quiz.

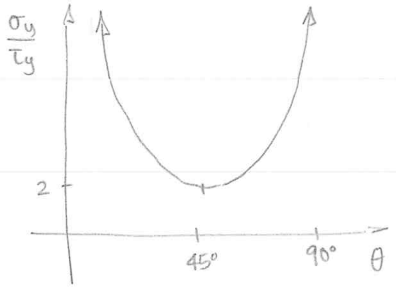
CE 323: QUIZ #2

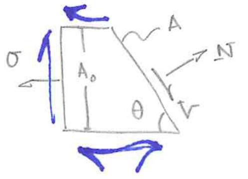
Catherine Hovell
October 9, 2003

1. metal - fails by shear



τ, τcrit, θ





$\frac{A_o}{A} = \sin\theta$

ΣF_x: $N\sin\theta + V\cos\theta = \sigma \cdot A_o$

ΣF_y: $V\sin\theta = N\cos\theta$

$\sigma A_o = V\frac{\sin^2\theta}{\cos\theta} + V\cos\theta \qquad \sigma A_o = \frac{V}{\cos\theta}$

$\frac{V}{A} = \tau_y \qquad \sigma_y = \frac{V}{A\sin\theta\cos\theta} \qquad \sigma_y = \frac{\tau_y}{\sin\theta\cos\theta}$

$\frac{\sigma_y}{\tau_y} = \frac{2}{\sin 2\theta}$

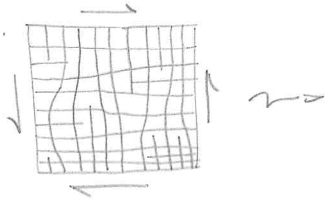
almost correct approach: FBD's; math a little suspect.

+3

a. strength of a material is derived from a pure sample of that material - no defects, dislocations, impurities, etc. Therefore failure occurs on a molecular level, by atoms being ripped apart. The strength of the test specimen is much less than the material strength, as the specimen is "real" - it has flaws that reduce the strength.

+3

c.



most dislocatio within the block are resolved through plastic displacements.

+5

11
B.

CE 323: QUIZ #2

2.

d = 6mm

E = 70 GPa

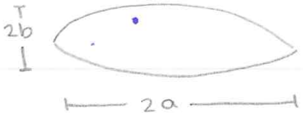
$G_c = 2\ J/m^2$

$\sigma_0 \sim \frac{E}{10}$ $\sigma_0 \sim \frac{70\ GPa}{10} = 7\ GPa$

P = 22 N

$$\sigma_\infty^{fail} = \frac{P}{A} = \frac{220\ N}{\pi/4(6\ mm)^2} = 7.78\times10^6\ Pa$$

assume eliptical flaw $\sigma_0 = \sigma_\infty^{fail}\left(1 + 2\frac{a}{b}\right)$



(a.)

$\frac{a}{b} \sim 450$

$$\left[\frac{7\times10^9\ Pa}{7.78\times10^6\ Pa} - 1\right]\frac{1}{2} = \frac{a}{b}$$

+2

$\frac{a}{b} = 450$

$\tau_0 = \frac{G}{\pi}$ $\tau_0 = \frac{2\ J/m^2}{3.14}$

$\tau_0 = 0.637\ J/m^2$

$2a = 160\ \mu m$

$\theta = 67.5°$

$$\sigma_0 = \sigma_\infty^{fail}\left(1 + 2\left(\frac{80\ \mu m}{450/80\ \mu m}\right)\right) = 7\ GPa$$

$\sigma_\infty^{fail} = 7.0\times10^9\ Pa$

$P = \sigma_\infty^{fail}\cdot A$ $A = \frac{\pi}{4}(0.006)^2$ +2

$P = 1.98\times10^5\ N$ $\boxed{P = 198\ kN}$

except this doesn't consider rotation of the slip plane, which would require a greater tensile stress, as only a percentage of that stress would cause shear strain in the right direction on the crack.

c. Energy absorption mechanisms during fracture of the chalk include sound wave propagation in the material, necking of the material before failure, and any kinetic energy the chalk pieces may obtain. Mainly, the strain energy between atoms in the chalk is converted to surface energy along a crack surface (which eventually passes through the entire piece of chalk).

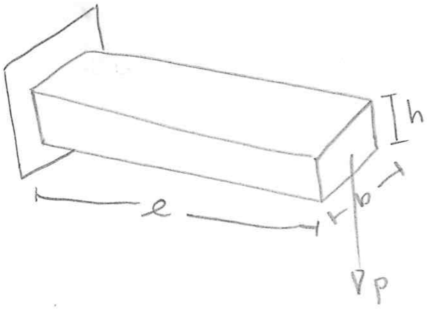
+4.

MATERIALS: MIDTERM

Catherine Hovell

October 28, 2003

1.



$\ell = 1m$

$b = 5cm$

$P = 50kN$

$h =$ variable

strong, light

$k = \frac{P_{max}}{\delta_{allow}}$

15
14
22
51

$$\delta = \frac{PL}{EA} = \frac{P\ell}{Ebh} \qquad \rho = \frac{m}{V} = \frac{m}{\ell b h}$$

$$\delta = \frac{P\ell}{E} \cdot \frac{1}{m/\rho\ell} = \frac{P\ell^2\rho}{Em}$$

$k = \frac{Em}{\ell^2\rho}$ want low m

$\frac{E}{\rho}$ large, $\frac{m}{\ell^2}$ sma

+6.

$$\sigma = \frac{P}{A} = \frac{P}{bh} = P\frac{\rho\ell}{m} = \varepsilon E$$

$$\frac{P_{max}}{A} = \sigma_{max} \propto \frac{E}{\rho}$$

$$\delta_{min} \propto \frac{\rho}{E}$$

ultimately, we want $\frac{\rho}{E}$ to be small

$k \propto \frac{E}{\rho}$

b. Balsa wood and nickel alloys lie along the same performance index line on a density vs. modulus material map. +4

c. Because the density of balsa wood is lower, more would be needed for the structure to be strong. In this case, h would be increased. For the alloy, the density is higher, so the same performance could be achieved with a smaller beam (smaller value for h). +5

15

CE 323: MIDTERM

2. a. Brittle materials fail when loaded by a large direct (normal) stress. Thus, to determine if a material will fail under a certain loading, the principal stresses and orientation should be found and compared to the allowable stresses. matl/bond rupture

+8

Ductile materials fail due to shear stress. A material will fail if the applied loads cause a shear stress at some orientation that is greater than the allowable stress. dislocation movement

c. The fracture toughnesses of ductile and brittle materials are different because for a brittle material, a stress state exists where all of the applied load goes to direct stress (along the principal axis). However, for a ductile material, only if $\sigma_x = -\sigma_y$ will there be an orientation where all of the applied load goes to shear stress. Thus, the load needed to fracture a ductile material is much larger than that needed to fracture a brittle material because only a portion of the applied stress lies along the plane where failure would occur.

+3

d. A higher fracture toughness means that the flaw would have to be larger to cause failure. conversely, a small flaw can cause failure in a material with a low fracture toughness.

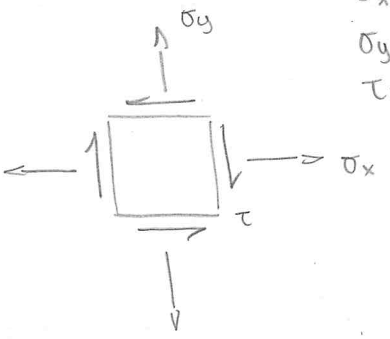
+3

⑭

MIDTERM O' JOY

3. $\sigma_y = 200$ MPa

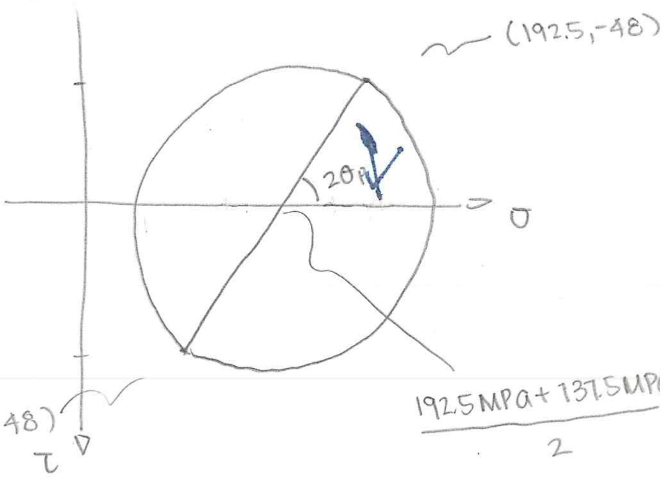
ductile



$\sigma_x = 192.5$ MPa

$\sigma_y = 137.5$ MPa

$\tau = -48$ MPa

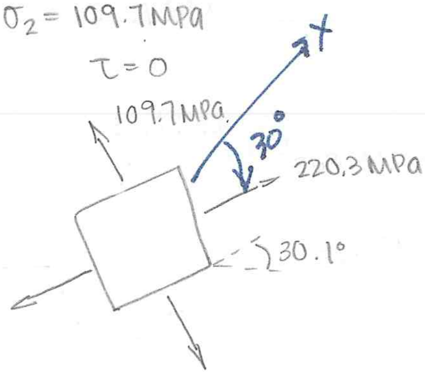


$\sigma_1 = 220.3$ MPa

$\sigma_2 = 109.7$ MPa

$\tau = 0$

(a.)



check angle

+9

$$\frac{192.5\,MPa + 137.5\,MPa}{2}$$

$= 165$ MPa

$$r = \sqrt{(\underbrace{13\quad}_{192.5MPa-165MPa} MPa)^2 + (-48MPa)^2}$$

$r = 55.3\,MPa = \tau_{max}$

$\sigma_{1,2} = 165\,MPa \pm 55.3\,MPa$

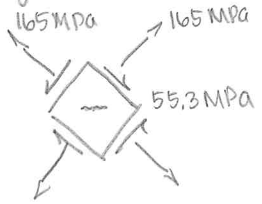
$\tau_{max} = 55.3$ MPa at $\sigma_x = \sigma_y = 165$ MPa

$$\tan 2\theta_p = \frac{48\,MPa}{192.5-165\,MPa} \qquad 2\theta_p = 60.2°$$

$\theta_p = 30.1°$

$$\tau_{abs} = \frac{\sigma_2}{2} = 110.2\,MPa$$

(b.) Yes, the material will yield at the given stress state. The orientation of the yield is at a 45° angle from the orientation of the principal axes, or 75.1° from the original orientation.



+4

Because $\tau_{abs} > \tau_{max\ inplane}$, the material will fail in the z-axis (out-of plane).

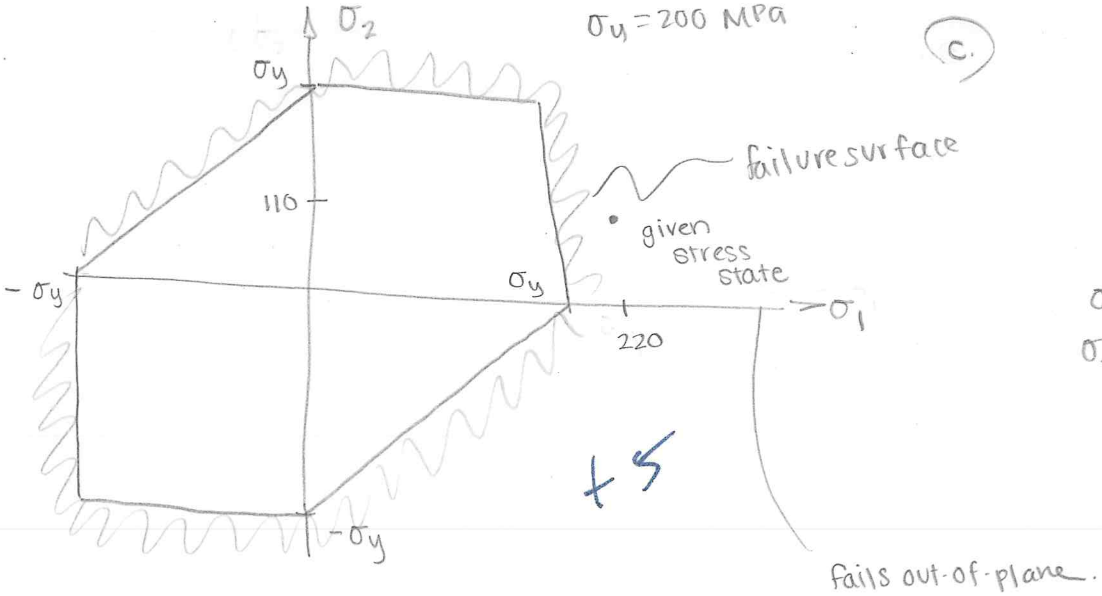
22

MATERIALS MIDTERM

3.

$\sigma_y = 200$ MPa

c.



$\sigma_1 = 220$ MPa

$\sigma_2 = 110$ MPa

+5

Von mises:

d.

$$\sigma_y = \frac{1}{\sqrt{2}}\sqrt{(\sigma_1-\sigma_2)^2+\sigma_1^2+\sigma_2^2}$$

$$\sigma_y = \frac{1}{\sqrt{2}}\left[(220\text{ MPa}-110\text{ MPa})^2+(220\text{ MPa})^2+(110\text{ MPa})^2\right]^{1/2}$$

$$\sigma_y = 190.5\text{ MPa}$$

$\sigma_{y\,von} < \sigma_{y\,given} < \sigma_1$ +2

Yes, the material will still yield. eh

If $\sigma_y = 200$ MPa and von mises is solved for σ_1, $\sigma_1 = 230.9$, which is > $\sigma_{1\,found}$. Therefore the von mises ellipse encloses the stress state given and the specimen does not fail.

Yes! (Smart check!) +2

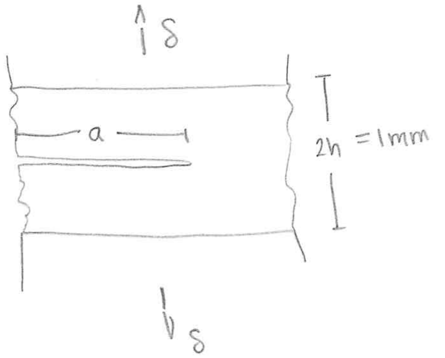
On my honor as a student, I have neither given nor received aid on this exam.

CE323: QUIZ #3

C.Hovell

46
43
89

2.



$a >> h$

$$K = \frac{E\delta}{\sqrt{(1-\nu^2)h}}$$

δ = displacement of one side

$E \sim 9\,GPa$

$\nu = 0.25$

$K_{Ic} \sim 0.5\ MPa \cdot m^{1/2}$

$\sigma_y = 85\ MPa$

crack advances when

$$\sigma_\infty^{fail} = \frac{K_{Ic}}{\sqrt{\pi a}}$$

yields when

$$\sigma_y = \frac{K_{Ic}}{\sqrt{2\pi r_p}}$$

$$a = \frac{1}{\pi}\left(\frac{K_{Ic}}{\sigma_\infty}\right)^2$$

$$K = K_{Ic}$$

$$K = \frac{E\delta}{\sqrt{(1-\nu^2)h}} = K_{Ic}$$

$$\delta = \frac{K_{Ic}\sqrt{(1-\nu^2)h}}{E}$$

$$\delta = \frac{(0.5\ MPa \cdot m^{1/2})\sqrt{(1-0.25^2)(0.5mm)}}{9\,GPa}$$

$$\delta = 1.2 \times 10^{-6}$$

a. 1.2 μm

+15

(b.) As the crack length increases, ~~a smaller~~ the same! stress is needed to propagate it. If all stress was removed, the crack would not continue, but the stress could simply be decreased and the flaw would grow (demanding a lower stress again, and causing more failure, etc.).

+3

to use LEFM, r_p must be small compared to other dimensions

$$r_p \sim \frac{1}{2\pi}\left(\frac{K_{Ic}}{\sigma_y}\right)^2$$

$$r_p \sim \frac{1}{2\pi}\left(\frac{0.5\,MPa \cdot \sqrt{m}}{85\,MPa}\right)^2 \sim 5.5\ \mu m$$

(c.) the h dimension is close to 100x as large as the plastic zone size, and $a >> h$, thus, it is acceptable to use LEFM.

+15

on my honor as a student I have neither given nor received aid on this quiz.

CE 323: QUIZ #3

Catherine Hovell
November 20, 2003

1.

(To quote you directly:)

The transistion flawsize is the size at which the failure stress predicted via linear elastic fracture mechanics (LEFM) is equal to the yield stress.

Thus, since

$$\sigma_f^{LEFM} = \frac{K_{Ic}}{\sqrt{\pi a}}, \quad \sigma_y = \frac{K_{Ic}}{\sqrt{\pi a_t}} \quad \text{or} \quad a_t = \frac{1}{\pi}\left(\frac{K_{Ic}}{\sigma_y}\right)^2$$

The transition flaw size can be used (as compared to flaw size) to determine which failure theory should be employed, and to give you an idea as to whether the specimen will yield or fracture with a given flaw.

If $a_{flaw} < a_t$, $\sigma_f^{LEFM} > \sigma_y$ $\rightsquigarrow$ yield before crack – use plastic yield theories

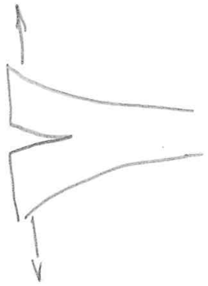
$\sigma_{flaw} > a_t$, $\sigma_f^{LEFM} < \sigma_y$ $\rightsquigarrow$ crack growth before yielding

└ use LEFM to analyze

Material properties:

$K_{Ic} \equiv$ toughness of a material and σ_y, right?!

$K_I = \sigma_\infty \sqrt{\pi a}$ where K is the stress intensity factor for a specimen failing via Mode I-opening



cracks occur when $K_I =$ critical value K_{Ic}

thus, $\sigma_\infty^{fail} \sqrt{\pi a} = K_{Ic}$

$\frac{46}{50}$

CE 323: QUIZ #3

C. Hovell

2 (cont'd).

(d.) Most likely, the ice between the steel plates is not pure. Thus, there are chunks of other crap woven into the ice lattice. Depending on what these chunks are, they could increase or decrease the toughness of the ice.

→ if particle is weak relative to the matrix, the particle could fail, leading to the failure of the matrix - cleavage failure

this has a greater liklihood of happening with larger particles.

The toughness and strength (and failure behavior) of the ice could change because of its very nature – the energy transfered into surface energy (among other things) has the ability to produce heat, which can melt the ice, causing a different surface due to the less structured form of melted (or melting) ice.

+8

e. $\sigma_{\infty} = \frac{K_{Ic}}{\sqrt{\pi a}}$ $K = \frac{E S}{\sqrt{(1-\nu^2)h}}$

If the ice were much thinner, the toughness (K_{Ic}) would be much higher, thus requiring a higher stress to be applied to cause crack growth.

One implication is the following scenario:

- ice gets into a gap, keeping two pieces apart
- crack develops
- water gets into new crack, freezes, pushing out on ice
- strip of ice is thicker, thus it takes less stress for new cracks to propagate, fill w/ water, and continue this cycle.

physically sound, just not quite what I asked for.

+2

(think frost heaving and the growth of cracks/potholes due to ice)

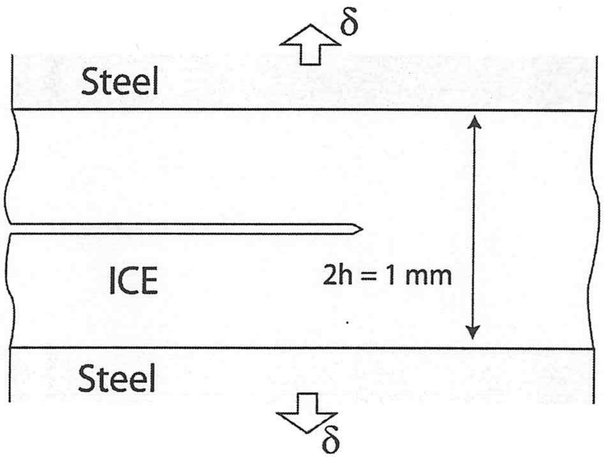
CE323 Properties and Behaviors of Materials	Quiz 3
Fall 2003	November 20th

Problem One:

Explain what is meant by the transition flaw size and how it is used in the analysis/design of structural components. Clearly identify the material properties that control the transition flaw size.

Problem Two:

A thin layer of ice fills the gap between two moving steel pieces of a drawbridge and bonds them together (ice sticks devilishly to steel). A crack that is much longer than the gap size (a >> h) is running through the center of the strip of ice, as shown below.



The stress intensity factor for this scenario is: $K = \dfrac{E\delta}{\sqrt{(1-\nu^2)h}}$, where δ is the displacement of one side. The elastic modulus of ice is $E \sim 9$ GPa. The Poisson's ratio is $\nu = 0.25$. The fracture toughness is $K_{Ic} \sim 0.5$ MPa m$^{1/2}$. The yield stress is $\sigma_y = 85$ MPa. The total gap size is: $2h = 1$ mm.

a. Compute the relative displacement of the two pieces of bridge needed to advance the crack.
b. If the two pieces of the bridge move just enough to start the crack and stopped, does the crack stop or continue one it begins to propagate – explain your answer.
c. Were you justified in using LEFM? Justify your answer with a quick calculation.
d. Are there any toughening mechanisms at work in this ice? Justify your answer.
e. What is the maximum tensile stress developed in the intact portion of the ice, far from the crack tip? What if the ice is much thinner, say 0.001 mm? Comment on the implications of your answer.

CE 323: HOMEWORK #1

~~47.5~~
$\frac{}{80}$

$\frac{60}{80}$

Catherine Hovell
September 12, 2003

1. $E = 0.5 \times 10^6$ psi

$\sigma_{max} = 6$ ksi

$\varepsilon_{max} = 1\% = 0.01$

P = 2.5 lb tension

$\varepsilon = \frac{\delta}{L_o}$ $\quad \sigma = \frac{P}{A}$ $\quad \sigma = E\varepsilon$

$\frac{P}{A} < 6$ ksi $\qquad \frac{P}{\frac{1}{4}\pi d^2} = E(0.01)$

$$d = \left[\frac{4\,(2.5\,\text{lb})}{\pi(0.01)(0.5\times10^6\,\text{psi})}\right]^{1/2}$$

$d = 0.0252$ in

check:

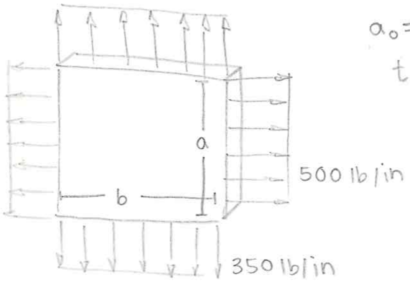
$$\frac{2.5\ \text{lb}}{\frac{1}{4}\pi(0.0252\,\text{in})^2} = 5000\ \text{lb/in}^2$$

$= 5$ ksi

$\boxed{d = 0.025\ \text{in}}$

✓10

2.



$a_o = b_o = 2$ in

$t_o = 0.25$ in

polystyrene

$E_P = 597 \times 10^3$ psi

$\nu = 0.25$

$$\boxed{\begin{array}{l} a' = 2.0030\,\text{in} \\ b' = 2.0055\,\text{in} \\ t' = 0.2496\,\text{in} \end{array}}$$

$\sigma_x = (500\,\text{lb/in})(a) / (a)(t)$

$\sigma_x = 2000\ \text{lb/in}^2$

$\sigma_y = (350\,\text{lb/in}) / (0.25\,\text{in})$

$\sigma_y = 1400\ \text{lb/in}^2$

$$\varepsilon_x = \frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} - \nu\cancel{\frac{\sigma_z}{E}}, \quad \varepsilon_y = \frac{\sigma_y}{E} - \nu\frac{\sigma_x}{E}, \quad \varepsilon_z = \frac{-\nu}{E}(\sigma_x + \sigma_y)$$

$$\varepsilon = \frac{\delta}{L_o} \qquad L' = L_o + \varepsilon L_o$$

$$\varepsilon_x = \frac{2000\,\text{psi}}{597\times10^3\,\text{psi}} - (0.25)\frac{1400\,\text{psi}}{597\times10^3\,\text{psi}} = 2.76\times10^{-3}$$

$$b' = b_o(1+\varepsilon_x) = 2\,\text{in}\,(1 + 2.76\times10^{-3})$$

$b' = 2.0055$ in

$$\varepsilon_y = \frac{1}{597\times10^3\,\text{psi}}\left[(1400\,\text{psi}) - (0.25)(2000\,\text{psi})\right]$$

$\varepsilon_y = 1.51\times10^{-3}$ $\qquad a' = (2\,\text{in})(1 + 1.51\times10^{-3})$

$a' = 2.00302$ in

$$\varepsilon_z = \frac{-0.25}{597\times10^{-3}\,\text{psi}}(2000\,\text{psi} + 1400\,\text{psi}) = -1.42\times10^{-3}$$

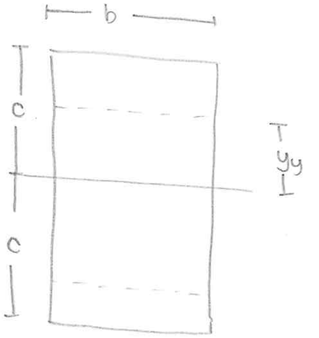
$$t' = (0.25\,\text{in})\left[1 + (-1.42\times10^{-3})\right]$$

$t' = 0.24964$ in

✓ 10

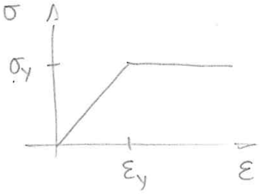
CE 323: HOMEWORK#1

3.



b=50 mm

C=60 mm



$M = 36.8$ kN·m

$\sigma_y = 240$ MPa

$E = 200$ GPa

$$\sigma = \frac{M_{max} \cdot Y}{I}$$

$$I = \tfrac{1}{12} b \cdot h^3$$

$$I = \tfrac{1}{12}(0.05\text{m})(0.12\text{ m}$$

$$I = 7.2\times10^{-6}\text{ m}^4$$

$$y = \frac{\sigma_y \cdot I}{M}$$

$$y = \frac{(240\times10^6\text{ Pa})(7.2\times10^{-6}\text{m}^4)}{36.8\times1000\text{ N}\cdot\text{m}}$$

$$\varepsilon = \frac{y}{\rho}$$

$y = 0.0470$ m

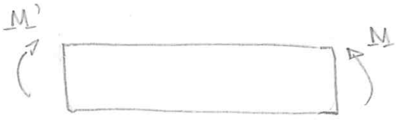
$\boxed{\text{elastic core thickness} = 93.9\text{ mm}}$ X

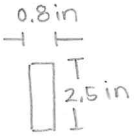
$$\varepsilon = \frac{\sigma_y}{E} = \frac{240\times10^6\text{ Pa}}{200\times10^9\text{ Pa}} = 0.0012$$

$$\rho = \frac{y}{\varepsilon} = \frac{0.0470\text{ m}}{0.0012} = 39.1$$

$\boxed{\rho = 39.1\text{m}}$ x +5

4.





$\sigma_y = 36$ ksi

$$\sigma = \frac{M \cdot y}{I} \qquad M = \frac{\sigma \cdot I}{y}$$

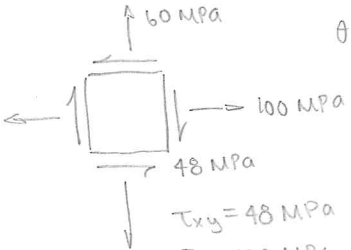
$$I = \tfrac{1}{12}(0.8\text{in})(2.5\text{in})^3$$

$$I = 1.042\text{ in}^4$$

$$M = \frac{(36\times1000\text{ lb/in}^2)(1.042\text{ in}^4)}{\tfrac{1}{2}(2.5\text{ in})}$$

$\boxed{M = 30\text{ kip}\cdot\text{in}}$ ✓10

5.



$\theta = 30°$

$\tau_{xy} = 48$ MPa

$\sigma_x = 100$ MPa

$\sigma_y = 60$MPa

$$\tan 2\theta_p = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2}$$

$$\theta_p = \frac{1}{2}\tan^{-1}\left[\frac{2\,(48\text{ MPa})}{100\text{MPa} - 60\text{MPa}}\right]$$

$\theta_p = 33.7°, 123.7°$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

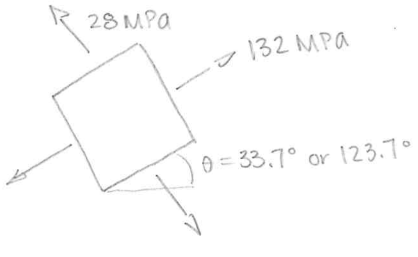
$$\sigma_x = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$\sigma_{1,2} = \frac{100\text{ MPa} + 60\text{ MPa}}{2} \pm \sqrt{\left(\frac{100\text{ MPa} - 60\text{ MPa}}{2}\right)^2 + (48\text{ MPa})^2}$$

$\sigma_1 = 132$ MPa

$\sigma_2 = 28$ MPa



$$\boxed{\begin{array}{l}\theta_p = 33.7°, 123.7° \\ \sigma_1 = 132\text{ MPa} \\ \sigma_2 = 28\text{ MPa}\end{array}}$$ ✓ 2½

$\sigma_{x'}, \sigma_{y'}, \tau_{x'y'}$ —?

+2½

continued...

CE 323: HOMEWORK #1

5 (cont'd).

$$\sigma_{x,y} = \frac{\sigma_x + \sigma_y}{2} \pm \frac{\sigma_x - \sigma_y}{2} \cos 2\theta \pm \tau_{xy} \sin 2\theta$$

$$\sigma_x = \frac{100\text{ MPa} + 60\text{ MPa}}{2} + \frac{100\text{ MPa} - 60\text{ MPa}}{2} \cos(60°) + (48\text{ MPa})\sin(60°)$$

$$\sigma_x = 131.6\text{ MPa}$$

$$\sigma_y = \frac{100\text{ MPa} + 60\text{ MPa}}{2} - \frac{100\text{ MPa} - 60\text{ MPa}}{2} \cos(60°) - (48\text{ MPa})\sin(60°)$$

$$\sigma_y = 28.4\text{ MPa}$$

$$\tau_{xy} = \frac{\sigma_y - \sigma_x}{2} \sin 2\theta + \tau_{xy} \cos 2\theta \qquad \tau_{xy} = \frac{60\text{ MPa} - 100\text{ MPa}}{2} \sin(60°) + (48\text{ MPa})\cos(60°) = 6.68\text{ MPa}$$

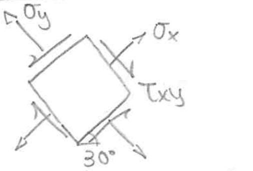
at 30° rotation:

$\sigma_x = 131.6$ MPa

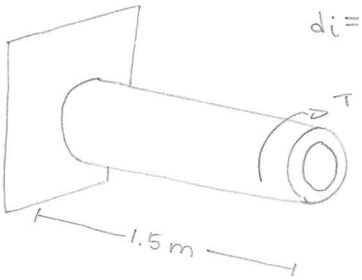
$\sigma_y = 28.4$ MPa

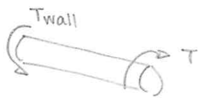
$\tau_{xy} = 6.68$ MPa

X



6.





$d_i = 40$ mm

$d_o = 60$ mm

$\tau_{max} = 120$ MPa

$$\tau_{max} = \frac{Tc}{J} \qquad \tau_{max} \text{ at } c = d_o$$

$$T = \frac{J\tau_{max}}{c} \qquad T = \frac{(1.02\times10^{-6}\text{ m}^4)(120\times10^6\text{ Pa})}{0.060\text{ m}}$$

$$J = \frac{\pi}{2}(c_o^4 - c_i^4)$$

$$J = \frac{\pi}{2}\left[\left(\frac{0.060\text{m}}{2}\right)^4 - \left(\frac{0.040\text{m}}{2}\right)^4\right]$$

$$J = 1.02\times10^{-6}\text{ m}^4$$

$$\boxed{T = 2.04\text{ kN}\cdot\text{m}}$$

✗

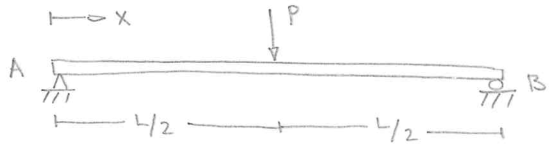
$$\tau(40\text{mm}) = \frac{(2.04\text{ kN}\cdot\text{m})(0.040\text{ m})}{1.02\times10^{-6}\text{m}^4}$$

$\tau_{min} = 80$ MPa
at c = 40 mm

✓ 5

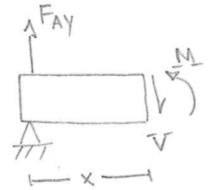
CE 323: HOMEWORK #1

7.



$F_{AY} = \frac{1}{2}P = F_{By}$

$F_{AX} = 0$



$V = F_{AY} = \frac{1}{2}P$

$M = F_{AY} \cdot x = \frac{1}{2}Px$

$$EI\frac{d^2v}{dx^2} = M(x) = \frac{1}{2}Px$$

$$EI\frac{dv}{dx} = \frac{1}{4}Px^2 + C_1 \qquad EIv = \frac{1}{12}Px^3 + C_1x + C_2$$

$\frac{dv}{dx}(L/2 = 0)$; $\qquad v(0) = 0$; $C_2 = 0$

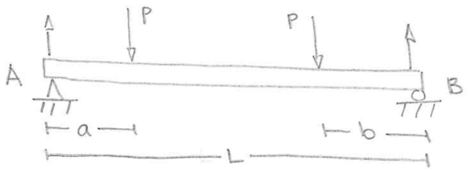
$$C_1 = -\frac{PL^2}{16} \qquad v = \frac{1}{EI}\left[\frac{1}{12}Px^3 - \frac{1}{16}PL^2x\right]$$

$$v(L/2) = \frac{1}{EI}\left[\frac{1}{12}P(L/2)^3 - \frac{1}{16}PL^2(L/2)\right]$$

$$\boxed{v(L/2) = \frac{PL^3}{48EI} \text{ down}}$$

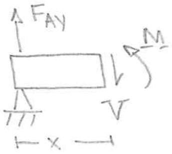
✓10

8.



a=b

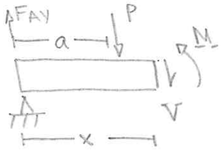
$F_{Ay} = F_{By} = P \qquad F_{Ax} = 0$



$M = F_{Ay} \cdot x = P \cdot x$

$EIv_1' = \frac{1}{2}Px^2 + C_1$

$EIv_1 = \frac{1}{3}Px^3 + C_1x + \cancel{C_2}$



$M = PX - P(x-a)$

$EIv_2' = Pax + C_3$

$EIv_2 = \frac{1}{2}Pax^2 + C_3x + \cancel{C_4}$

$C_3 = -Pa(L/2)$ as $v'(L/2) = 0$

$C_2, C_4 = 0$ as $v(0) = 0$

$v_1(a) = v_2(a)$

$\frac{1}{3}Pa^3 + C_1a = \frac{1}{2}Pa^3 - \frac{1}{2}Pa^2L$

$C_1 = \frac{1}{6}Pa^2 - \frac{1}{2}PaL$

$EIv_1 = \frac{1}{3}Px^3 + \left(\frac{1}{6}Pa^2 - \frac{1}{2}PaL\right)x$

$EIv_2 = \frac{1}{2}Pax^2 - \frac{1}{2}PaLx$

at load points:

$EIv_1 = \frac{1}{3}Pa^3 + (1/6\,Pa^2 - 1/2\,PaL)a$

$EIv_1 = \frac{1}{2}Pa^3 - \frac{1}{2}Pa^2L$

$$\boxed{v_{\text{load points}} = \frac{Pa^2}{2EI}[a - L]}$$

✗

at midpoint:

$EIv_2 = \frac{1}{2}Pa(L/2)^2 - \frac{1}{2}PaL(L/2)$

$EIv_2 = \frac{1}{8}PaL^2 - \frac{1}{4}PaL$

$$\boxed{v_{mid} = \frac{PaL}{4EI}\left[\frac{1}{2}L - 1\right]}$$

✗

+5

Catherine Hovell
September 18, 200

CE 323: HOMEWORK #2

26/35

Problem One:

1. $W = \frac{1}{2}\sigma\varepsilon$ W = strain energy density

$W = \frac{1}{2}\sigma\frac{\sigma}{E}$ $W_{max} = \frac{1}{2}\sigma_f^2 \cdot \frac{1}{E}$

$$W_{max} = \frac{\sigma_f^2}{2 \cdot E}$$

$M \propto W_{max}$, $\boxed{M = \frac{1}{2}\frac{\sigma_f^2}{E}}$ +5

looking at a E vs. σ_f materials map, knowing the relationship σ_f^2/E...

+ 3

2. Engineering ceramics and elastomers maximize σ_f and E ??

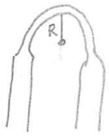
3. along the $\sigma_f^2/E = c$ line, brick and silicone show similarities, as the square of their failure strength over their moduli result in similar constants. +3

4. cost and durability are two examples of important factors not included in the performance index. They can be used to eliminate potential materials.

+3

14/20

Problem Two:

1. 

$R = \frac{t}{2}\left(\frac{E}{\sigma_f}\right) = \frac{t}{2}\left(\frac{1}{\varepsilon_f}\right)$

$\delta = \frac{PL^3}{EI}$ $\varepsilon = y/R$ $y = \frac{1}{2}t$

$\varepsilon = t/2R$

$R = t/2\varepsilon = \frac{t}{2}\left(\frac{E}{\sigma_f}\right)$

+5

2. $\frac{\sigma_f}{E}$ = constant

- elastomers allow for the greatest allowable strain without (or before) failure
- engineering ceramics would also have a higher σ_f and modulus, although the allowable strain would be less

+3

3. cost should come into play in considering these two situations. Also, expected life span and problems caused by failure. A shampoo bottle will only be expected to last on the order of a few months, and can still be used when the hinge is broken. Thus, a cheaper, weaker product can be used. on the micro-scale mirror, if the allowable strain can be less, it would probably be worth the money for the extra strength and durability. This is important to consider in light of the expected life span of the mirror and the problems caused if the hinge fails.

+4

12/15

CE 323: HOMEWORK #3

Catherine Hovell
October 3, 2003

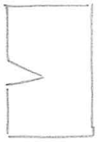
29/30

1. strain energy → surface energy

$S.E. = V \cdot \frac{1}{2}\sigma\varepsilon = \frac{1}{2} s \cdot e \cdot x$

e = strain
s = stress/strength

$e = \frac{s}{E}$



$\frac{S.E.}{m^2} = \frac{s^2 \cdot x}{2E} = 2G$ $\quad s = 2\sqrt{\frac{G \cdot E}{x}}$

$s = \sigma$
$G = \gamma$

$\boxed{a.\ \sigma_0 = 2\left(\frac{\gamma \cdot E}{x}\right)^{1/2}}$

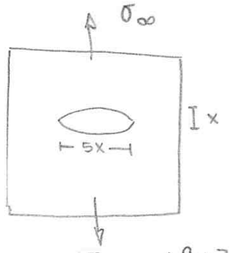
+3

$\gamma = 1\ J/m^2 = G$
$E = 400\ GPa$
$a = 3\ \text{Å} = x$

$s = 2\left[\frac{(1\ J/m^2)(400\ GPa)}{3 \times 10^{-10} m}\right]^{1/2}$

$\boxed{b.\ \sigma_0 = 73.0\ GPa}$

+3



a = 5b
b = b

$\sigma_{max} = \sigma_\infty\left[1 + 2\left(\frac{a}{b}\right)\right] = \sigma_0$ at failure

$\sigma_0 = \sigma_\infty^{fail}\left[1 + 2\left(\frac{5b}{b}\right)\right] = 73.0 GPa$

$\sigma_\infty^{fail} = \frac{73.0\ GPa}{1 + 2(5)} = 6.64\ GPa$

$\boxed{c.\ \sigma_\infty^{fail} = 6.64\ GPa}$

+3

$\sigma_\infty^{fail} = 1\ GPa, \quad 2a = 1 \times 10^{-6} m, \quad \rho = ?$

$\sigma_{max} = \sigma_\infty^{fail}\left[1 + 2\sqrt{\frac{a}{\rho}}\right] = \sigma_0$

$\left[\frac{1}{2}\left(\frac{\sigma_0}{\sigma_\infty^{fail}} - 1\right)\right]^2 = \frac{a}{\rho}$

$\frac{a}{\rho} = \frac{1}{4}\left(\frac{73.0\ GPa}{1\ GPa} - 1\right)^2$

$\rho = \frac{\frac{1}{2} \times 10^{-6} m}{2592}$

$\boxed{d.\ \rho = 1.93\ \text{Å}}$

+3

$2a = 1\ \mu m$
$\rho = 10\ nm$
$\sigma_\infty^{fail} = 100\ GPa$

$\sigma_0 = \sigma_\infty^{fail}\left(1 + 2\sqrt{\frac{a}{\rho}}\right)$

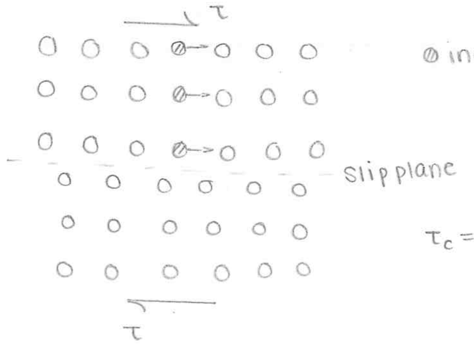
$\sigma_0 = (100\ GPa)\left[1 + 2\left(\frac{1 \times 10^{-6} m}{10 \times 10^{-9} m}\right)^{1/2}\right]$

$\boxed{e.\ \sigma_0 = 2100\ GPa}$

+3

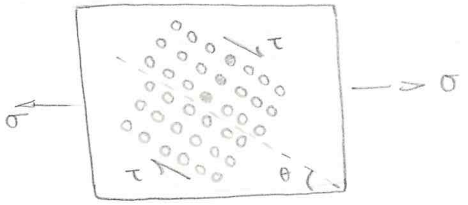
CE 323: HOMEWORK #3

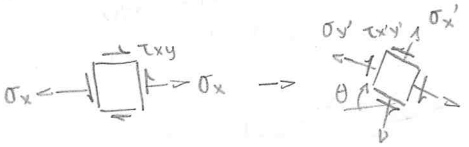
2.



⊘ indicate dislocation

τ_c = critical shear stress





Minimum tensile yield stress occurs at θ = 45°

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$\sigma_y = \frac{2\tau_c}{\sin 2\theta} \qquad \frac{d\sigma_y}{d\theta} = 0 \text{ at min } \sigma_y \text{ angle}$$

$$\tau_{x'y'} = \frac{-\sigma_x}{2}\sin 2\theta \qquad -\sigma_x = \sigma_y \text{ as defined in problem}$$

$$\frac{d\sigma_y}{d\theta} = \tau_c \ln(\sin 2\theta) \qquad \frac{d\sigma_y}{d\theta} = 0 \text{ at } \ln(\sin 2\theta) = 0$$

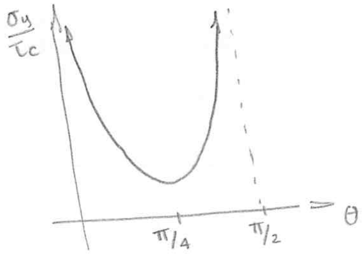
$$\sin 2\theta = 1,\ \theta = \pi/4$$

$$\tau_{x'y'} = \frac{\sigma_y}{2}\sin 2\theta \quad \text{or} \quad \frac{2}{\sin 2\theta} = \frac{\sigma_y}{\tau_c}$$

$$\boxed{\text{b. } \sigma_{y_{min}} = 2\tau_c \text{ at } \theta = \pi/4}$$ +5

$$\boxed{\text{a. } \tau_c = \frac{\sigma_y}{2}\sin 2\theta}$$

+5



asymptotal at 0, π/2

c. The theoretical yield stress perpendicular and parallel to the direction of loading is infinite, since sin(0)=0 and sin(π)=0, and stress is inversely related to sin2θ. Trying to calculate σ_y at those points results in a value divided by zero, which is infinite, and therefore impossible in the real world.

+4

CE 323/363: HOMEWORK #4

Catherine Hovell
October 27, 2003

95

$$G = -\frac{1}{b}\frac{dU}{da} = -\frac{1}{b}\frac{d}{da}(S.E. + W_{ext})$$

S.E. = Strain energy in the beam

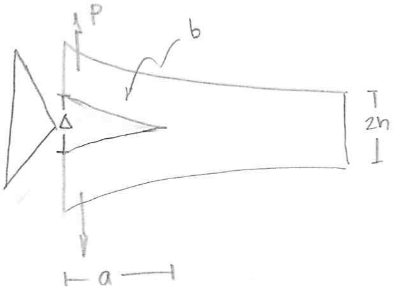
W_{ext} = external work done by loading mech.

1. $S.E. = \frac{2P^2a^3}{Ebh^3}$

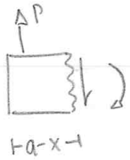
$\Delta = \frac{4Pa^3}{Ebh^3}$

$$\frac{SE}{vol.} = \frac{1}{2}\sigma\cdot\varepsilon$$

$$SE = \frac{1}{2}\frac{\sigma^2}{E}dV = \frac{1}{2}\frac{\sigma^2}{E}dx\,dy\cdot b$$



$$SE = \int_{-\frac{h}{2}}^{h/2}\int_0^a \frac{1}{2}E\left[\varepsilon(x,y)\right]^2 b\,dx\,dy$$



$V = P \quad N = 0$

$M = P(a-x)$

$\sigma = \frac{My}{I} \quad \varepsilon = \frac{P(a-x)y}{EI}$

$$SE = \frac{1}{2}\frac{P^2}{EI^2}\int_{-\frac{h}{2}}^{h/2}\int_0^a (a-x)^2 y^2 b\,dx\,dy$$

$$SE = \frac{P^2 b}{2EI^2}\int_{-\frac{h}{2}}^{\frac{h}{2}}\left[\frac{-1}{3}(a-x)^3\cdot y^2\Big|_0^a\right]dy$$

$$SE = \frac{P^2 b}{2EI^2}\int_{-\frac{h}{2}}^{\frac{h}{2}}\left[\frac{1}{3}a^3y^2\right]dy = \frac{P^2 b}{2EI^2}\cdot\frac{1}{3}a^3\left(\frac{1}{3}y^3\Big|_{-\frac{h}{2}}^{h/2}\right)$$

$$SE = \frac{1}{2}\cdot\frac{1}{3}\cdot\frac{1}{3}\frac{P^2 b}{EI^2}a^3\left(\frac{h^3}{8}+\frac{h^3}{8}\right) = \frac{1}{72}\frac{P^2ba^3h^3}{EI^2}$$

$$I = \frac{1}{12}b(h)^3 = \frac{bh^3}{12}$$

$$\boxed{SE = \frac{2P^2a^3}{Ebh^3}}$$

+25

$$\frac{d^2\Delta}{dx^2} = \frac{M(x)}{EI} = \frac{P(a-x)}{EI}$$

$$\frac{d\Delta}{dx} = \frac{P}{EI}\left(ax - \frac{1}{2}x^2\right)$$

$$\Delta = \frac{P}{EI}\left(\frac{1}{2}ax^2 - \frac{1}{6}x^3\right) = \frac{P}{EI}\cdot\frac{1}{3}a^3 = \frac{Pa^3}{E}\cdot\frac{1}{3}\cdot\frac{12}{bh^3}$$

$$\boxed{\Delta = \frac{4Pa^3}{Ebh^3}}$$

C_1, C_2 will $= 0$

because $\Delta, v' = 0$ at $x = 0$

+25

CE 323/363: HOMEWORK #4

2. $G = -\frac{1}{b}\frac{dU}{da} = -\frac{1}{b}\frac{d}{da}(SE + W_{ext})$ $\quad SE = \frac{2P^2a^3}{Ebh^3}$ $\quad \Delta = \frac{4Pa^3}{Ebh^3}$ $\quad W_{ext} = -P\Delta$

load control: $dP = \text{constant}$

$$G = -\frac{1}{b}\left[\frac{d}{da}\left(\frac{2P^2a^3}{Ebh^3}\right) - \frac{d}{da}\left(\frac{4Pa^3}{Ebh^3}\right)P\right]$$

$$G = -\frac{1}{b}\left[\frac{6P^2a^2}{Ebh^3} - \frac{12P^2a^2}{Ebh^3}\right] = \frac{6P^2a^2}{Eb^2h^3}$$

$$\boxed{G = \frac{6P^2a^2}{Eb^2h^3}}$$

+25

displacement control: $d\Delta = \text{constant}$

$W_{ext} = 0$ $\quad \frac{\Delta Ebh^3}{4a^3} = P$

0

$$G = -\frac{1}{b}\left[\frac{d}{da}\left(\frac{2\Delta^2E^{\cancel{2}}b^{\cancel{2}}h^{\cancel{6}\,3}\cancel{a^3}}{\cancel{E}\cancel{b}h^{\cancel{3}}\cdot 16a^{\cancel{6}\,3}}\right) - \cancel{\frac{d}{da}\left(\frac{\Delta Ebh^3}{4a^3}\right)\Delta}\right]$$

0

$$G = -\frac{1}{b}\left[\frac{\Delta^2Ebh^3}{8}\frac{d}{da}(a^{-3}) - \cancel{\frac{\Delta^2Ebh^3}{4}\frac{d}{da}(a^{-3})}\right] \qquad \frac{d}{da}a^{-3} = -3a^{-4}$$

$$G = -\frac{1}{b}\left[-\frac{3}{8}\frac{\Delta^2Ebh^3}{a^4} + \frac{3}{4}\frac{\Delta^2Ebh^3}{a^4}\right]$$

$$\boxed{G = \frac{3\Delta^2Eh^3}{8a^4}}$$

+20

C.Hovell

Ah, A New Life of Materials

August 28, 2003

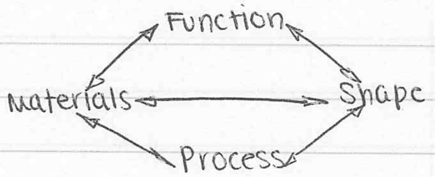
Functionality
- safety
- cost
- durability
- market need

Shape
- aesthetics
- environment
- dimensions

Material
- ceramics / glass
- metal
- polymers / elastomers
- wood / biological

All comes down to PROCESS
- shape : individual members, macrostructure
  - e.g. can it be done? change properties?

Central Design Process is an interaction



material selection without shape

$k_{des} \equiv$ design stiffness $= \frac{P_{max}}{\delta_{allow}} \equiv$ specified

$$d = \left(\frac{64\,K_{des}}{3\pi E}\right)^{1/4} \qquad m = \frac{\ell}{4}\left(\frac{64\pi K_{des}\,\ell^3}{3}\right)^{1/2} \underline{\underline{\frac{\rho}{E^{1/2}}}}$$

Key Factor

minimize $\boxed{\rho/\sqrt{E}}$ to get maximum stiffness with minimum weight

↑

PERFORMANCE INDEX

Materials Selection in Mechanical Design,
by M.F. Ashby, Butterworth-Heinemann Pubs, London, 1996

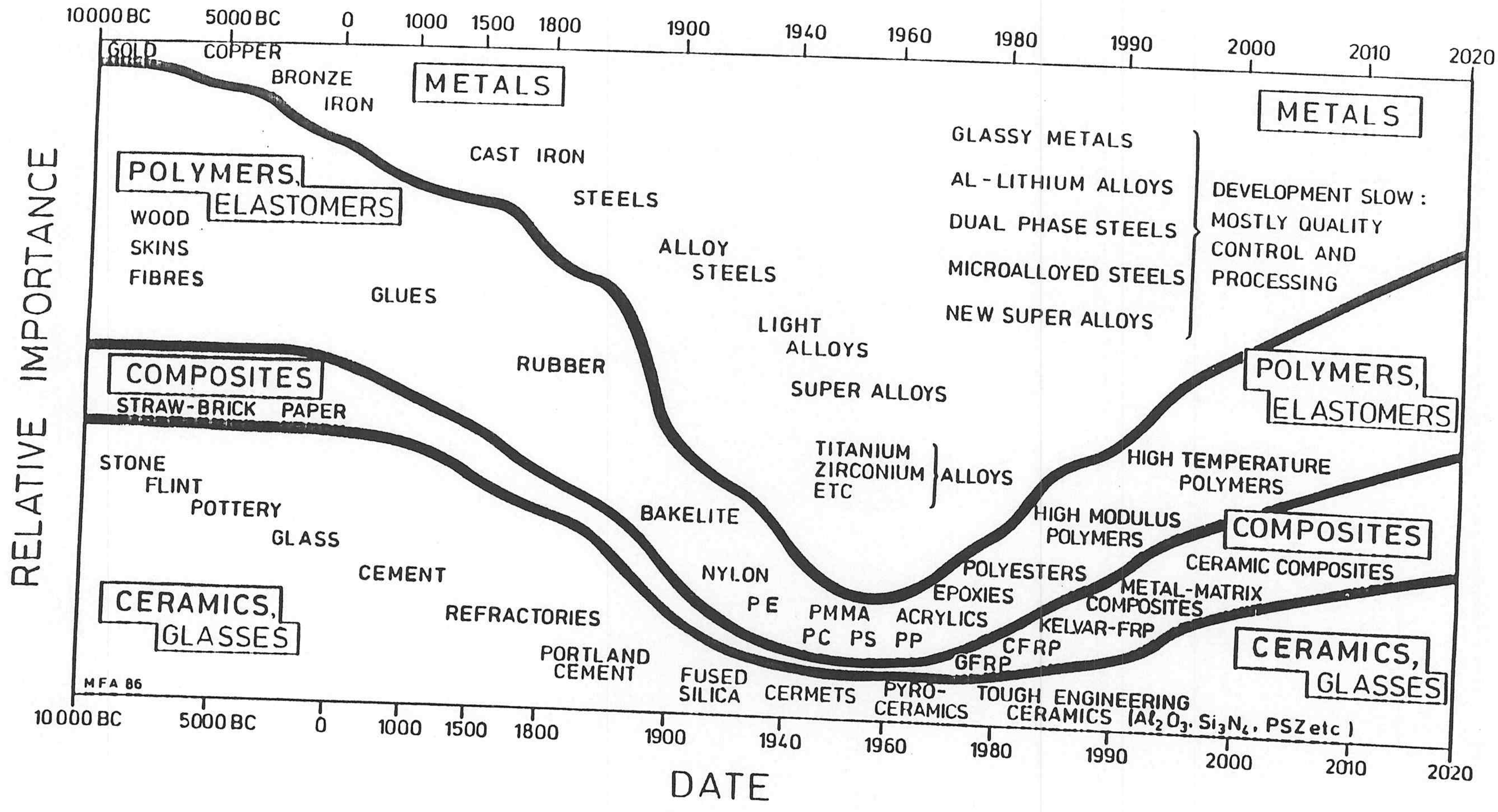
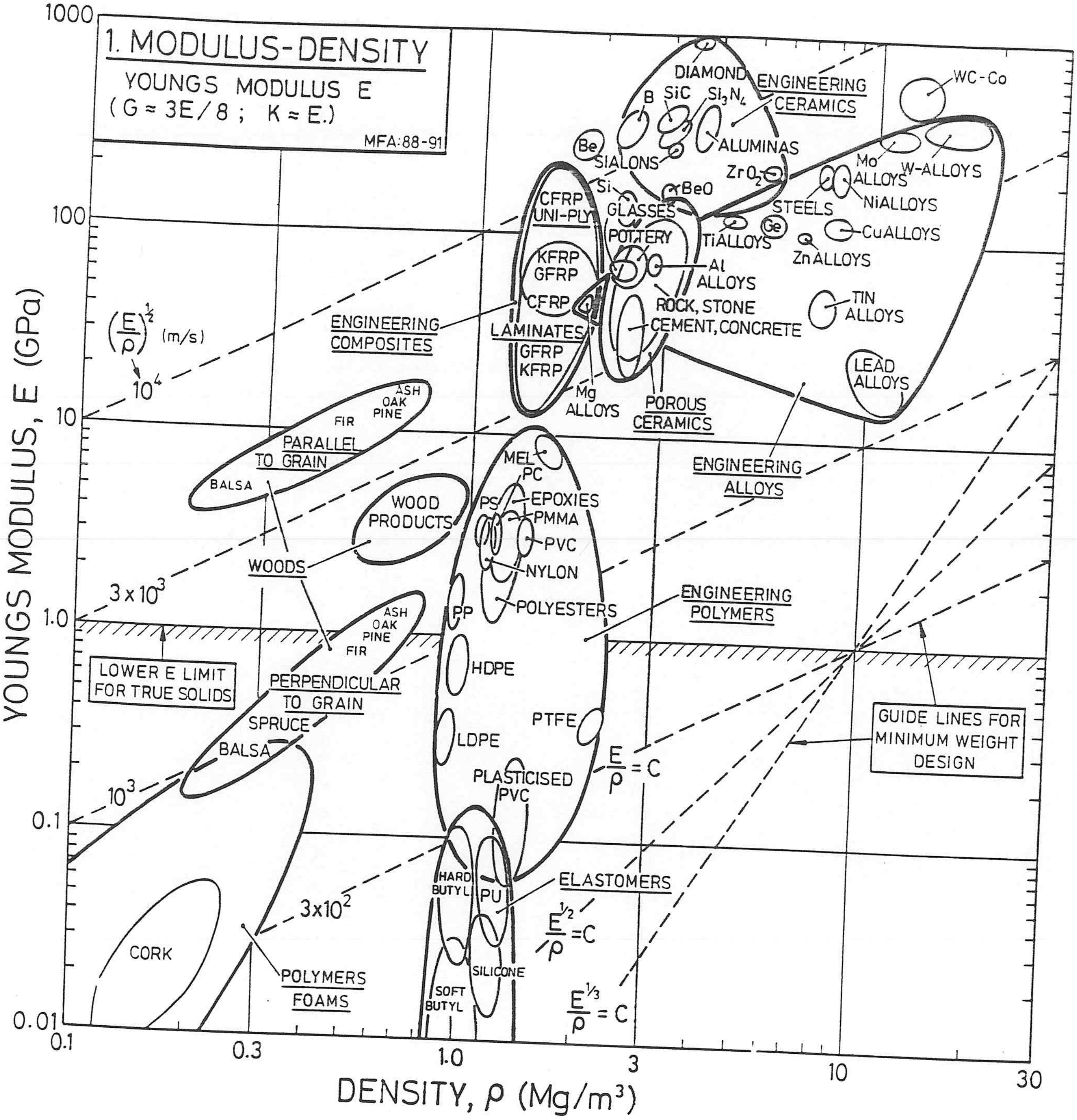
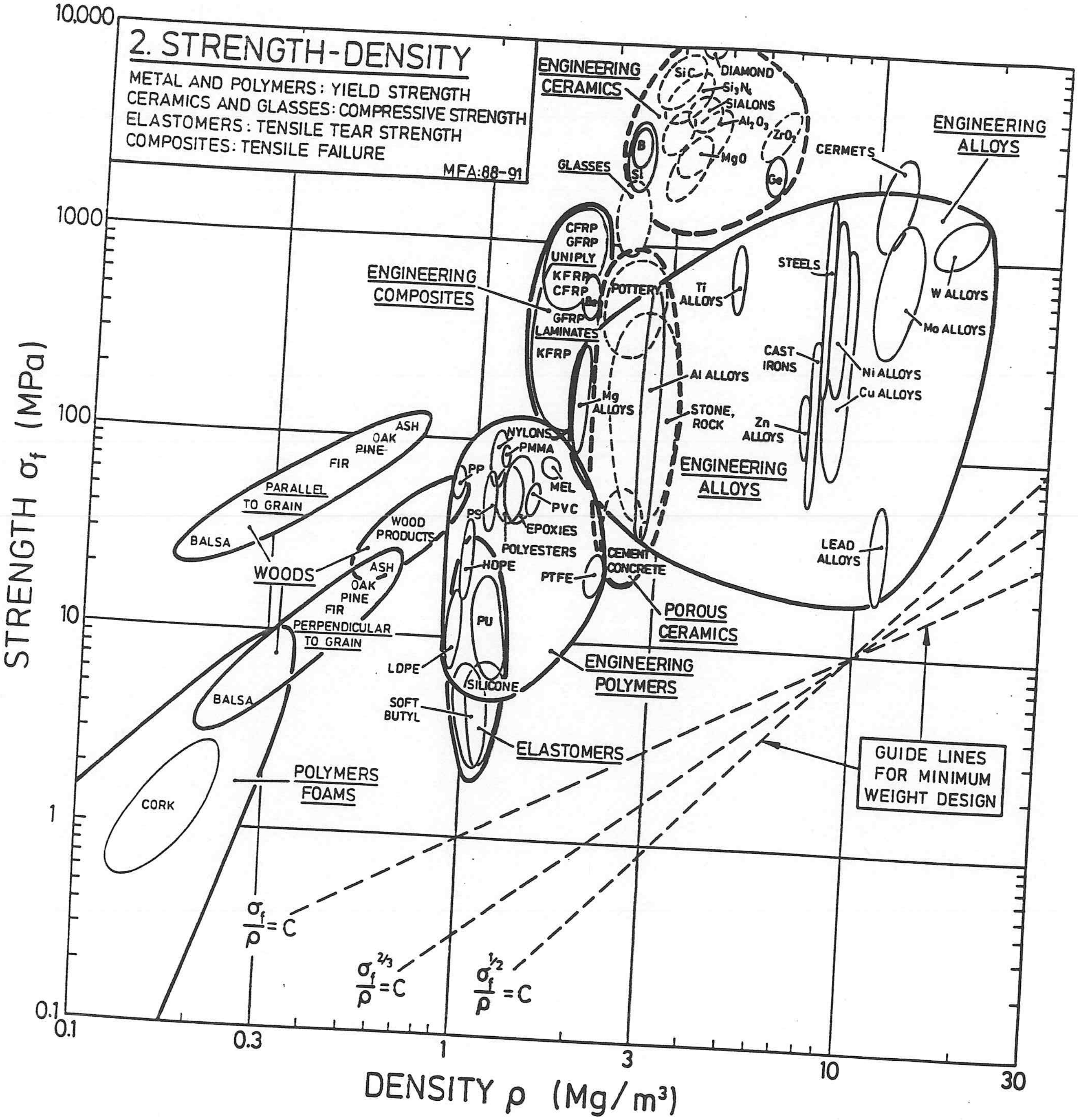
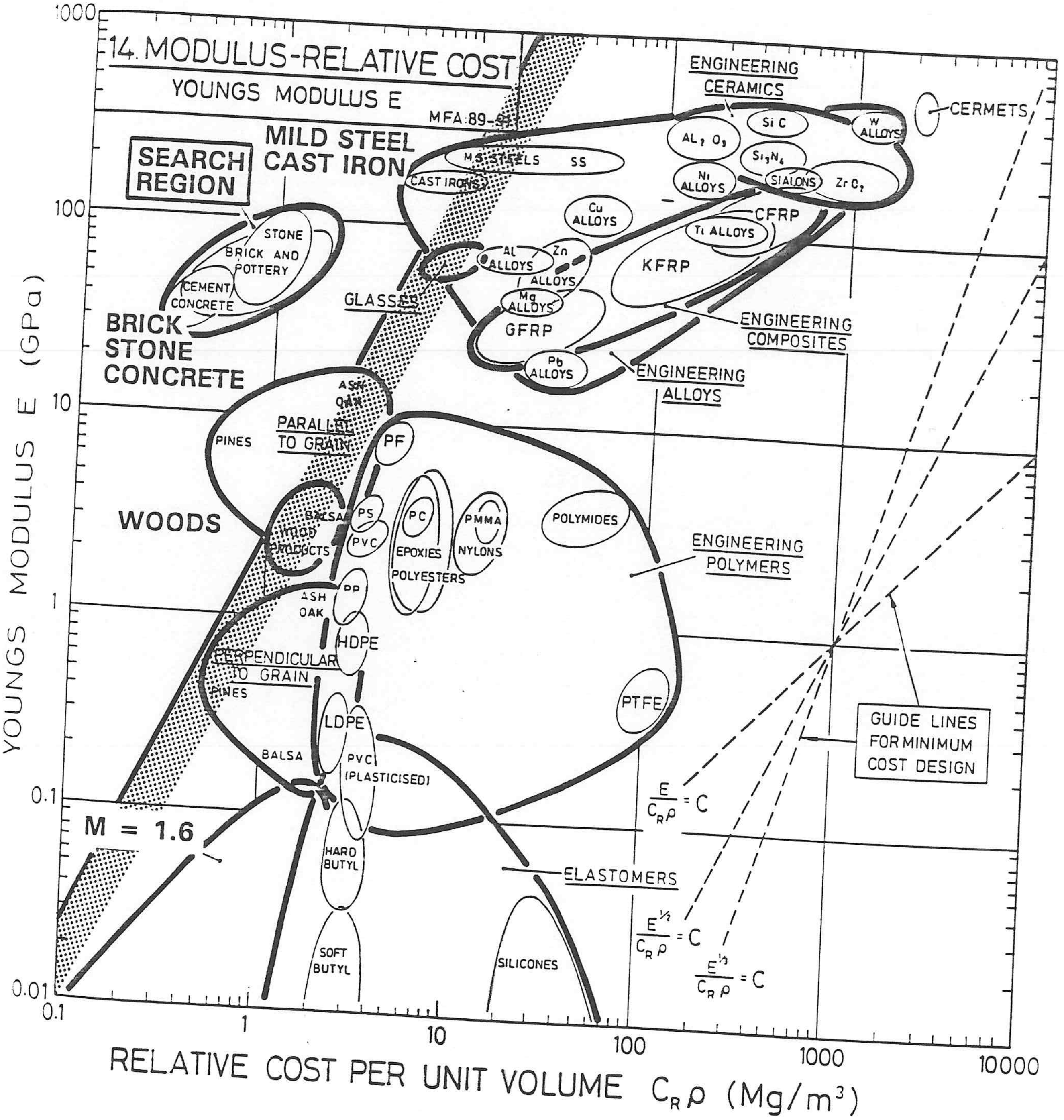


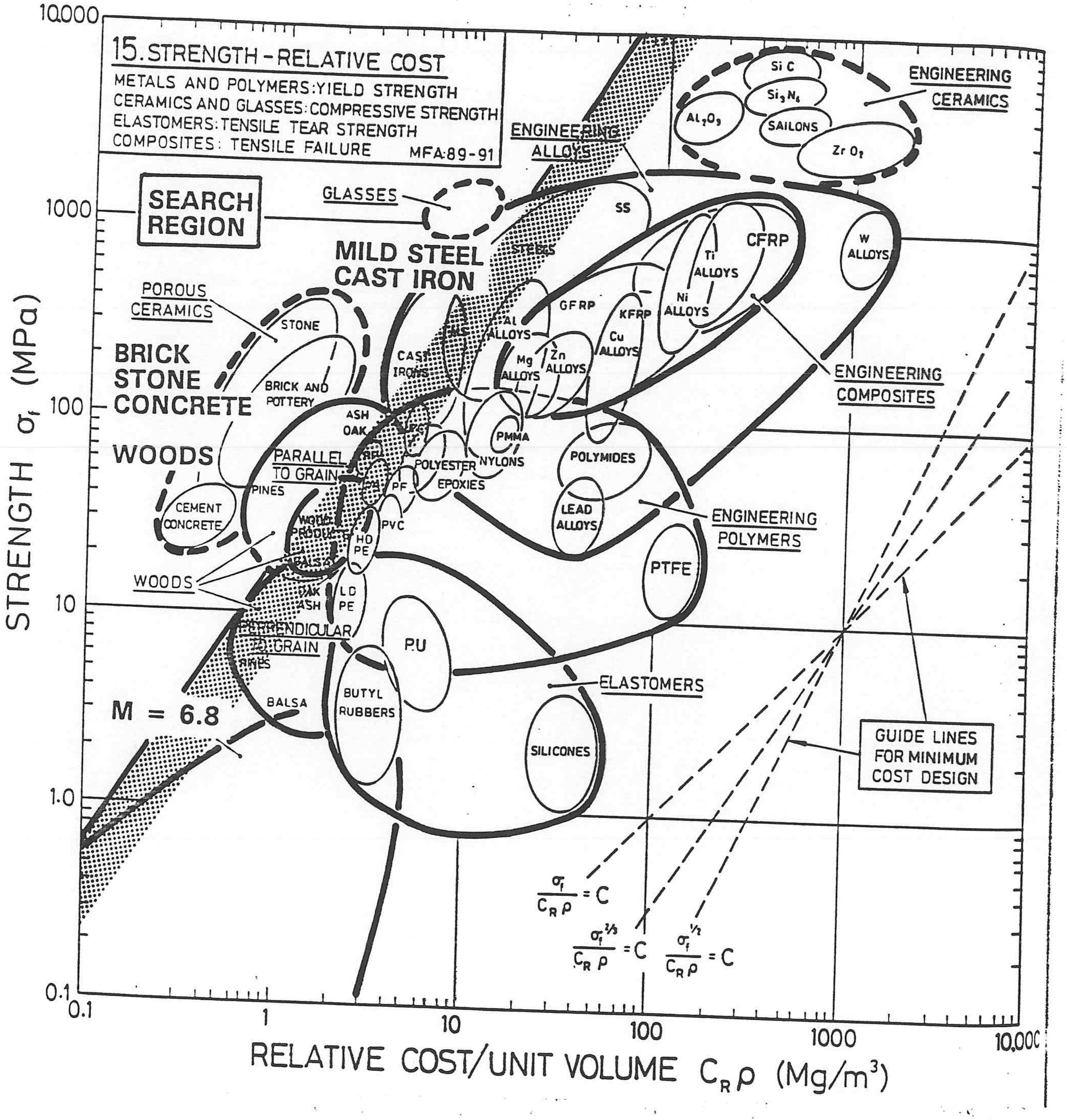
FIG. 1.1 The evolution of engineering materials.



FRP = fibre reinforced plastic


2. STRENGTH-DENSITY
METAL AND POLYMERS: YIELD STRENGTH
CERAMICS AND GLASSES: COMPRESSIVE STRENGTH
ELASTOMERS: TENSILE TEAR STRENGTH
COMPOSITES: TENSILE FAILURE
MFA:88-91
ENGINEERING CERAMICS
SiC
DIAMOND
Si_3N_4
SIALONS
Al_2O_3
ZrO_2
MgO
B
Si
Ge
GLASSES
CERMETS
ENGINEERING ALLOYS
W ALLOYS
Mo ALLOYS
STEELS
Ti ALLOYS
CAST IRONS
Ni ALLOYS
Cu ALLOYS
Al ALLOYS
STONE, ROCK
Zn ALLOYS
ENGINEERING ALLOYS
LEAD ALLOYS
CFRP
GFRP
UNIPLY
KFRP
CFRP
Be
GFRP
LAMINATES
KFRP
POTTERY
ENGINEERING COMPOSITES
Mg ALLOYS
NYLONS
PMMA
PP
MEL
PS
PVC
EPOXIES
POLYESTERS
HDPE
PTFE
CEMENT
CONCRETE
POROUS CERAMICS
PU
ENGINEERING POLYMERS
LDPE
SILICONE
SOFT BUTYL
ELASTOMERS
ASH
OAK
PINE
FIR
PARALLEL TO GRAIN
BALSA
WOOD PRODUCTS
WOODS
ASH
OAK
PINE
FIR
PERPENDICULAR TO GRAIN
BALSA
CORK
POLYMERS FOAMS
GUIDE LINES FOR MINIMUM WEIGHT DESIGN
$\frac{\sigma_f}{\rho} = C$
$\frac{\sigma_f^{2/3}}{\rho} = C$
$\frac{\sigma_f^{1/2}}{\rho} = C$
STRENGTH σ_f (MPa)
10,000
1000
100
10
1
0.1
DENSITY ρ (Mg/m³)
0.1
0.3
1
3
10
30


14. MODULUS-RELATIVE COST
YOUNGS MODULUS E
MFA 89-91
SEARCH REGION
MILD STEEL
CAST IRON
M.S. STEELS
SS
CAST IRONS
ENGINEERING CERAMICS
Si C
AL$_2$ O$_3$
Si$_3$N$_4$
W ALLOYS
CERMETS
Ni ALLOYS
SIALONS
Zr C$_2$
Cu ALLOYS
CFRP
Ti ALLOYS
KFRP
STONE
BRICK AND POTTERY
CEMENT
CONCRETE
BRICK
STONE
CONCRETE
GLASSES
Al ALLOYS
Zn ALLOYS
Mg ALLOYS
GFRP
Pb ALLOYS
ENGINEERING COMPOSITES
ENGINEERING ALLOYS
ASH
OAK
PARALLEL TO GRAIN
PINES
PF
WOODS
BALSA
WOOD PRODUCTS
PS
PVC
PC
EPOXIES
POLYESTERS
PMMA
NYLONS
POLYMIDES
ENGINEERING POLYMERS
PP
ASH
OAK
HDPE
PERPENDICULAR TO GRAIN
PINES
LDPE
PTFE
BALSA
PVC (PLASTICISED)
HARD BUTYL
SOFT BUTYL
SILICONES
ELASTOMERS
M = 1.6
GUIDE LINES FOR MINIMUM COST DESIGN
$\frac{E}{C_R\rho} = C$
$\frac{E^{1/2}}{C_R\rho} = C$
$\frac{E^{1/3}}{C_R\rho} = C$
1000
100
10
1
0.1
0.01
YOUNGS MODULUS E (GPa)
0.1
1
10
100
1000
10000
RELATIVE COST PER UNIT VOLUME $C_R\rho$ (Mg/m^3)

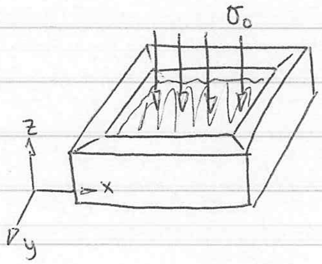

15. STRENGTH - RELATIVE COST
METALS AND POLYMERS: YIELD STRENGTH
CERAMICS AND GLASSES: COMPRESSIVE STRENGTH
ELASTOMERS: TENSILE TEAR STRENGTH
COMPOSITES: TENSILE FAILURE
MFA:89-91
SEARCH REGION
GLASSES
ENGINEERING ALLOYS
ENGINEERING CERAMICS
Si C
Si3 N4
Al_2O_3
SAILONS
Zr O2
SS
STEELS
CFRP
W ALLOYS
Ti ALLOYS
Ni ALLOYS
MILD STEEL CAST IRON
POROUS CERAMICS
STONE
BRICK STONE CONCRETE
BRICK AND POTTERY
WOODS
CAST IRONS
GFRP
KFRP
Al ALLOYS
Mg ALLOYS
Zn ALLOYS
Cu ALLOYS
ENGINEERING COMPOSITES
ASH
OAK
PARALLEL TO GRAIN
PMMA
NYLONS
POLYMIDES
POLYESTER
EPOXIES
PF
PS
PVC
PINES
CEMENT CONCRETE
WOOD PRODUCTS
LEAD ALLOYS
ENGINEERING POLYMERS
HD PE
BALSA
WOODS
OAK ASH
LD PE
PTFE
PERPENDICULAR TO GRAIN
PINES
P.U
ELASTOMERS
BUTYL RUBBERS
BALSA
M = 6.8
SILICONES
GUIDE LINES FOR MINIMUM COST DESIGN
$\frac{\sigma_f}{C_R\rho} = C$
$\frac{\sigma_f^{2/3}}{C_R\rho} = C$
$\frac{\sigma_f^{1/2}}{C_R\rho} = C$
10,000
1000
100
10
1.0
0.1
0.1
1
10
100
1000
10,000
STRENGTH σ_f (MPa)
RELATIVE COST/UNIT VOLUME $C_R\rho$ (Mg/m³)

C. Hovell

It's Kelly's Birthday

September 2, 2003

Last semester Review (whee!)



$\sigma = \frac{P}{A}$ $\varepsilon = \frac{\Delta}{l_0}$

$\sigma = E\varepsilon$ Hooke's Law

$\sigma_z = \sigma_0$

$$\begin{cases} \varepsilon_x = \frac{\sigma_x}{E} - \frac{\nu}{E}(\sigma_y + \sigma_z) \\ \varepsilon_y = \frac{\sigma_y}{E} - \frac{\nu}{E}(\sigma_x + \sigma_z) \\ \varepsilon_z = \frac{\sigma_z}{E} - \frac{\nu}{E}(\sigma_y + \sigma_x) \end{cases}$$

$\varepsilon_y = \varepsilon_x = 0$ if $E_{conc} \gg E_{fill}$ (which it is)

$$\sigma_x = \sigma_y = \left(\frac{\nu}{1-\nu}\right)\sigma_0$$

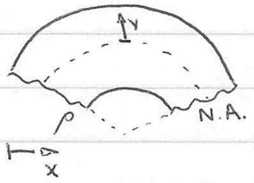
if $\nu = 0.5$, material is said to be $\underbrace{\text{incompressible}}$

$\varepsilon_z = 0$ ↳ no elastic volume change

Rubber comes close!

APPARANTLY BRIDGES ARE BUILT ON TOP OF RUBBER "POT BEARINGS" THAT EXPERIENCE THIS.

Bending



$\varepsilon(x,y) = K(x) \cdot y = \frac{y}{\rho}$

curvature

radius of curvature

⊳ $\varepsilon = 0$ along neutral axis

⊳ ε varies linearly in y direction

$I = \int_{area} y^2 \, dA$

$$\boxed{\sigma = \frac{My}{I}}$$

curvature: $\frac{d^2v}{dx^2} = \frac{M(x)}{E \cdot I}$

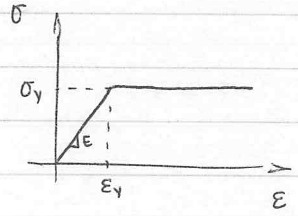
$\delta = \frac{PL^3}{3EI}$

C.Hovell

Argh. Early Morning AND Two Pages

September 2,2003

Elastic / Plastic Bending



elastic / perfectly plastic bending

> once a certain stress is achieved, strain can be almost any number

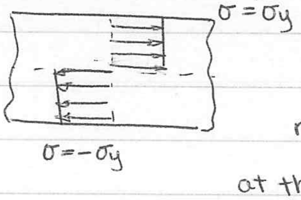
> fails from the outside first

$$\boxed{\varepsilon = y/\rho}$$

is <u>always</u> true!

- plastic skin – outer edge where ε_y, σ_y have been reached
- elastic core – inner section where $\sigma_x < \sigma_y$

Plastic LIMIT:



no elastic core

at this stage, M applied = Mmax

plastic collapse:

$$M_p = \int_{bot}^{top} b\,\sigma(y)\,y\,dy = 2\int_0^{h/2} \sigma_y \cdot y \cdot b\,dy$$

$$\boxed{M_p = \frac{b\,\sigma_y h^2}{4}}$$ rectangular cross-section

deflections could be huge, but no failure

$$\underline{M} = \int \left(\sigma(y)\right) dA \cdot y = \int y \cdot \sigma(y) \cdot b \cdot dy$$

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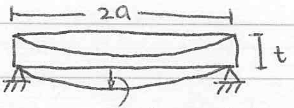
Substitute! whee!

September 4, 2003

Material selection



δ, due solely to the moss of the beam

ex. 2a = 6 m (diameter)

$m = \pi a^2 t \rho$

weight = 70 tons $\quad t = 1\,m$

$\delta = \frac{3}{4\pi} \frac{mga^2}{Et^3}$

cost = $200 million

RAR. MY ABILITY TO FOCUS IS SO IN THE NEGATIVE ATTENTION SPAN RANGE. DAMN RAIN!

back to Material selection

$\frac{1}{m} \propto \frac{E^{1/3}}{\rho}$

- weight
- durability
- cost! $\quad$ cost $\propto m^2$, or volume

Design of structures

stiff, strong, CHEAP! [cost / unit volume] → C_R

Performance indecies

- Bending $\quad M = \frac{E^{1/2}}{\rho C_R}$
- Buckling (same eq.)

$P_{CRIT} = F_S P_{REAL} = \frac{\pi^2 E I}{\ell^2}$

(considers safety factor)

$$COST = 2(F_S F)^{1/2} \left(\frac{\ell^4}{2\pi}\right)^{1/2} \frac{\ell C_R}{E^{1/2}} = \boxed{2\left(\frac{S_F \cdot F}{2\pi E}\right) \ell^2 C_R}$$

Strong + Cheap

Failure by tension

$$\sigma_{max} = \frac{M_{max} \cdot r}{I}$$

C. Hovell

QUIZ #1 NOW NEXT WEEK

September 9, 2003

Quiz #1 Topics:

- strength of materials
  - hooke's law
  - bending
- elastic / plastic bending
- stiffness vs. strength
  (in book, chapt. 1, 2)
- material property definitions
- material selection
- Gordon chapters 1 and 2

MAN, JUST SHOOT ME NOW, PLEASE.

Atomic bonds

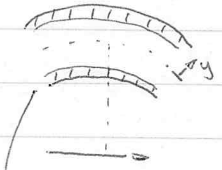
ionic, covalent, ~~mat~~ metallic, hydrogen

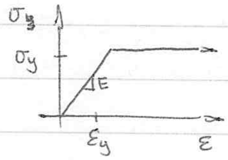
glass can be stronger than metal because hydrogen, ionic bonds are stronger than metallic bonds

Strength

- > flaw tolerance : strength
- > toughness : ductility
- > stiffness : modulus, due to bonding, structure

Homework problems / help



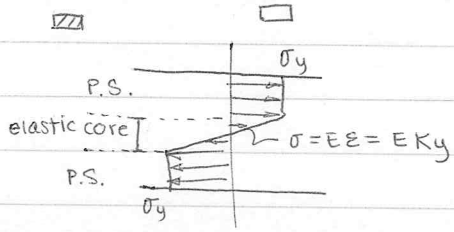


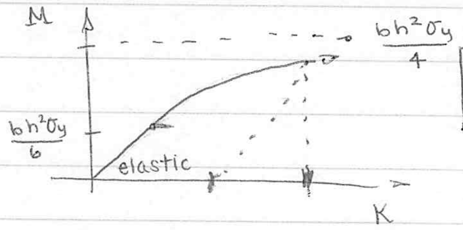
σ_y = allowable elastic stress (yield stress)

$\varepsilon(x,y) = K \cdot y$ MAN I SUCK AT KAPPAS.

> always acceptable to use as long as assumptions hold
> - plane sections remain plane
> - small deformations
> - plane sections remain ⊥ to N.A. which experiences no strain

> plastic skin (with elastic core)



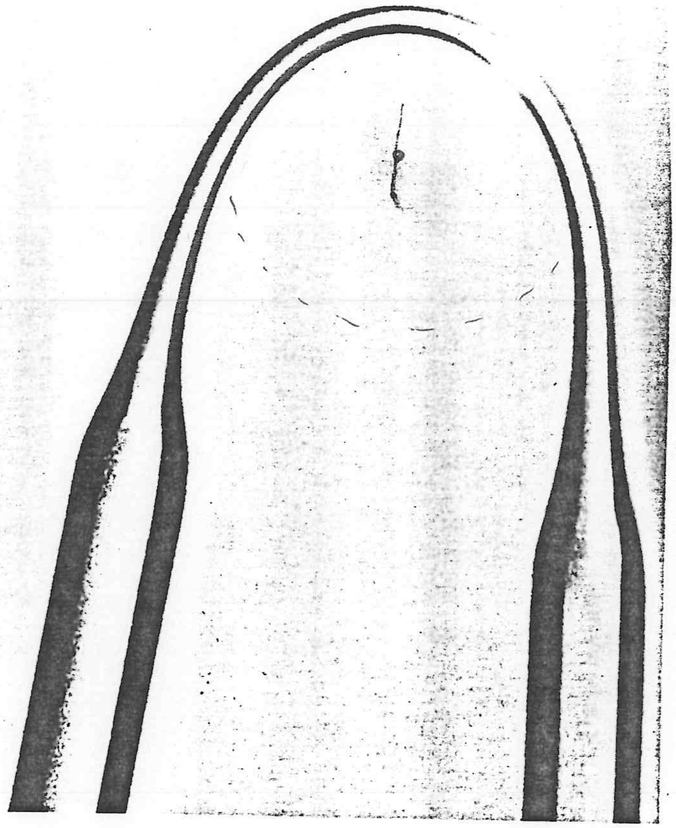


$$\boxed{K = \frac{\sigma_y}{E} \cdot \frac{1}{c} = \frac{\varepsilon_y}{c}}$$

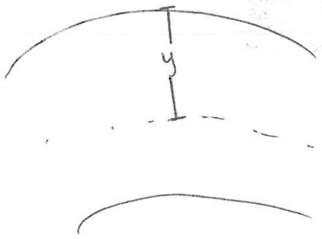
c = distance from N.A. to edge of elastic core

From: Strongest Man Competition

- how strong is this bar?
- what is it made of?



- no plasticity, so top is narrow due to original shape, not necking
- start with a FBFD wait, no.



$\varepsilon(y) = K \cdot y = y/\rho$

$y = \frac{1}{2} d$

ρ can be written in terms of d

$\varepsilon_{max}(y) = \frac{1/2\, d}{\rho(d)}$

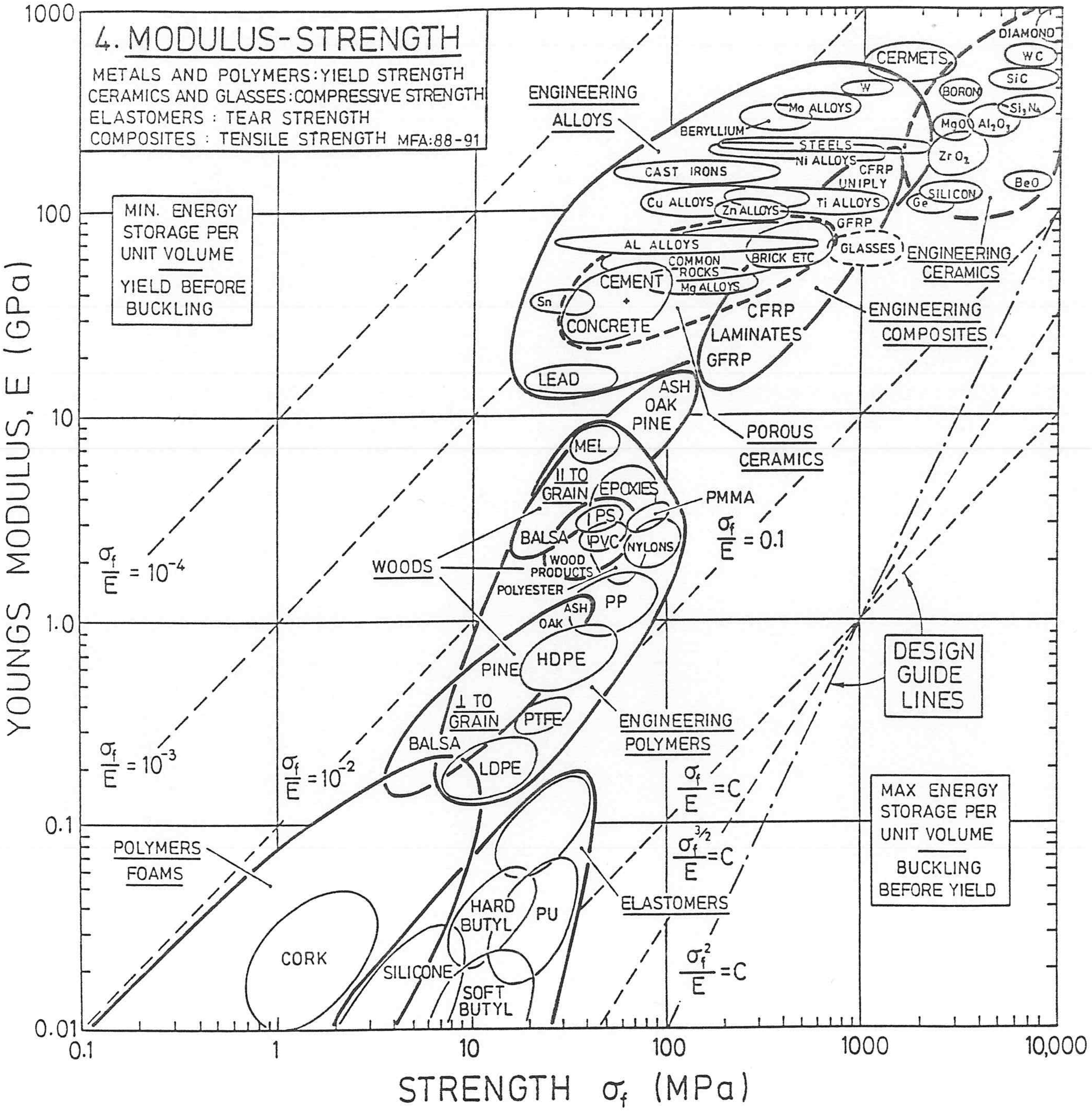
made of glass!

E = 50 GPa-ish

σ_f = 5000 MPa

what makes something "strong"?

- Small enough to have no impurities within it
- if defects were present, it would break
- to break in half, you need to break the bonds between atoms!



less strain allowable ← ε → more strain allowable

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Sigh September 11, 2003

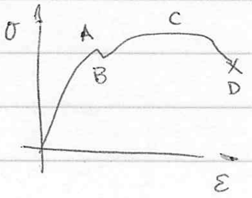
Jello

flexible, weak, brittle

Strength vs. stiffness vs. toughness

(BTW, I DON'T THINK I'VE EVER TAKEN SO FEW NOTES IN BEGLEY'S CLASS EVER!)

- stiff vs. flexible (stiffness)
  - lb/in, N/m relates forces and displacements, or, stress and strain
- strength (strong vs. weak)
  - psi = lb/in², Pa = N/m² maximum load carried by a given area, or, maximum stress



"before bad things happen"

A: last point of recoverable deformation, σ_y

C: ultimate (tensile) strength, σ_{uts}

D: fracture stress, strength, σ_{frac}

σ_f, failure strength… made up term, essentially – no specific point

- toughness (tough vs. brittle)
  - related to ductility, deformation before failure
  - the ability of a material to withstand flaws or defects
  - related to the energy required to make new surfaces i.e. advance a crack

Important points:

$k \equiv P/\delta$ $E^{1/2}/\rho$ to maximize stiffness

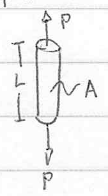
C. Hovell

I Feel SO SICK. And, Stupid Hurricane. September 18, 2003

Back to Three Keys of Solid Mechanics

- Properties of Materials
    - constitutive descriptions
    - $\sigma - \varepsilon$ relationships – Hooke's Law
- Displacements and strain
    - kinematics (description of motion)
- Equilibrium

Example #1



$\delta = \frac{PL}{EA}$

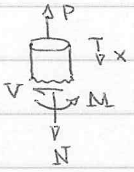
$\sigma = E\varepsilon \quad \sigma = \frac{F}{A}$

$\Sigma F_x = V = 0 \quad \tau = 0$

$\Sigma F_y : P = N$

$\Sigma M : M = 0$

$\varepsilon = \frac{\delta}{L}$



$\tau = \frac{V}{A}$

$\boxed{\varepsilon = Ky = \frac{d^2v}{dx^2}y}$

→ for a uniform strain

known because of uniform F

($\varepsilon = \frac{du}{dx}$ for more general situations)

similar to springs: P vs. δ, slope = $\frac{AE}{L}$

F vs. δ, slope = k

$\frac{AE}{L} = k = \frac{P}{\delta}$

Energy in springs

equal to work done on spring

$dW = F\,dx = kx\,dx \qquad \boxed{W = \frac{1}{2}kx^2} \qquad \text{or } W = \frac{1}{2}k\delta^2$

$W = \frac{1}{2}\frac{AE}{L}\delta^2$ = strain energy

strain energy/volume = $\frac{1}{2}\frac{E}{L^2}\delta^2 \quad \boxed{\frac{1}{2}E\left(\frac{\delta}{L}\right)^2 = \frac{1}{2}E\varepsilon^2 = \frac{1}{2}\sigma\varepsilon}$

Material comparison

$AE = \frac{PL}{\delta}$ as P, L, δ generally are givens, A varies with changes in E due to differing materials

strain is set, it's practically given

treat actual springs in terms of length, diameter etc. due to wraps?

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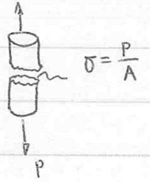
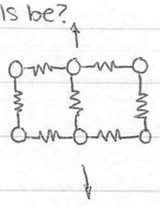
Ah: That QUIZ went Better

September 23, 2003

Strength of solids

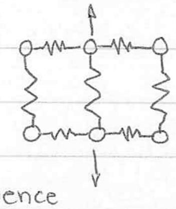
how strong should materials be?





chemical, physical bonds on the atomic level

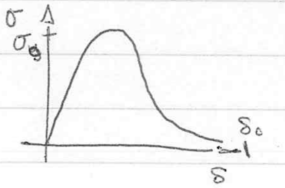
what energy is needed to break these bonds?

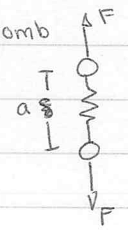


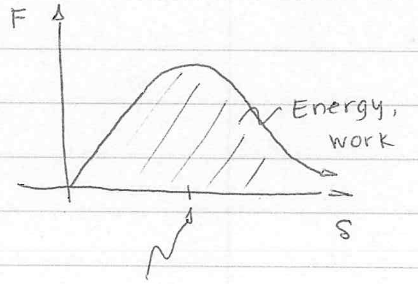
stretched (deformed), but has not yet failed. calculable δ

- Failure mechanism: sequence of physical events that lead to failure

eg. an atomic bomb







point at which interaction stops/slows because they're too far apart

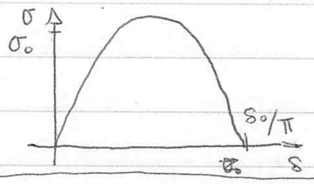
$\sigma_0 \equiv$ theoretical strength

$\delta_0 \equiv$ critical separation (no more interaction)

area under F-δ curve is the work of separation

Ah, a derivation I don't like but is likely to come up on a test.

assume stress-separation to be like a sine curve



$$\sigma = \sigma_0 \sin\left(\frac{\pi\delta}{\delta_0}\right)$$

for $\delta \ll \delta_0$,

$$\sigma = \sigma_0 \frac{\pi\delta}{\delta_0} = E\varepsilon = \varepsilon_0 \frac{\pi\delta_0}{\delta_0}$$

BUT- PART DOES NOT FAIL JUST BY RIPPING BONDS APART! unless completely pure

$$E\frac{\delta}{a} = \sigma_0 \frac{\pi\delta}{\delta_0}$$

$a \equiv$ equilibrium interatomic spacing (basically, original length)

$$\boxed{\sigma_0 = \frac{E\delta_0}{\pi a}}$$

stiff things are strong

so, what about $\frac{\delta_0}{a}$? δ_0 = when there's no interaction

$E/10 < \sigma_0 < \cancel{10}\ E$ δ_0/a is between 1/3 x and 3x

gives strength, from modulus, on ~~ao~~ a range of one order off... OK, something

C.Hovell

Missed Notes October, etc, 2003

Theoretical strength

$\sigma_0 = \frac{E\delta_0}{\pi a}$ δ_0=critical separation

a = atomic (spacing?) $\gamma_{H_2O} = 0.75\ J/m^2$

strain energy/unit volume = $\frac{1}{2}\sigma\varepsilon$

per unit area = $\frac{1}{2}\sigma\varepsilon a = 2\gamma$

$\boxed{\sigma_0 = 2\sqrt{\frac{\gamma E}{a}}}$ $\sigma_0 \simeq \frac{E}{10}$

So, shear stress:

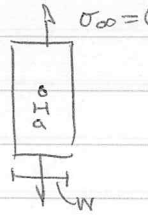
$\tau_0 \propto f(G,a)$ $G = \frac{E}{2(1+\nu)}$ $\tau(\delta) = \tau_0 \sin\left(\frac{\pi\delta}{a}\right) = \tau_0 \sin(\pi\gamma)$

for small strains,

$\frac{d\tau}{d\gamma} = G$ $\boxed{\tau_0 = \frac{G}{\pi}}$

Defects: holes and cracks

assume a<<w - really small holes

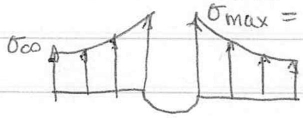


strength w/o hole = σ_0 CIRCLE!

$P = \sigma_\infty \cdot 2w \cdot t$

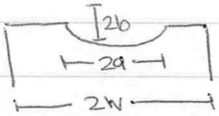
average stress on section w/ hole = $\frac{P}{(2w-2a)t}$

$\sigma_{avg} = \frac{\sigma}{1 - a/w} = \sigma_0$ at failure $\frac{\sigma_\infty^{fail}}{\sigma_0} = 1 - \frac{a}{w}$ for flaw-insensitive, ductile materials



3=stress concentration factor

Elliptical hole



$\sigma_{max} = \sigma_\infty\left(1 + 2\left(\frac{a}{b}\right)\right)$ if $a=0$, $\sigma_{max} = \sigma_\infty = \sigma_0$

if $b=0$, $\sigma_{max} = \infty$, $\sigma_\infty^{fail} = 0$!

radius of the tip:

$\boxed{\rho_{tip} = \frac{b^2}{a}}$ stress concentration factor $= 1 + 2\left(\frac{a}{b}\right)$

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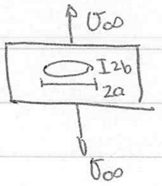
Tuesday Pre-My Arrival September 30, 2003

Theoretical Strength

- bond rupture
- strain energy —> surface energy $\sigma_a = E/10$, $\tau_a = E/10$
- atomic sliding via shear

Flaws, Defects



$$\sigma_{max} = \sigma_\infty \left(1 + 2\frac{a}{b}\right) = \sigma_\infty \left(1 + 2\sqrt{\frac{a}{\rho}}\right)$$

failure happens when

— maximum stress in part (panel) hits the theoretical strength of material

$$\sigma_{max} = \sigma_0 = \sigma_\infty^{fail} \left(1 + 2\frac{a}{b}\right)$$

σ_∞^{fail} = remote stress at failure

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FUCK! I Hate Life.

September 30, 2003

Failure happens when...

- maximum stress in part hits the theoretical strength of the material

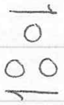
$$\sigma_{max} = \sigma_0 = \sigma_\infty^{fail}\left(1 + 2\, {}^{a}/_{b}\right).$$

$$\sigma_\infty^{fail} = \frac{\sigma_0}{1 + 2\sqrt{{}^{a}/_{\rho_{tip}}}}$$ remote stress at failure   a = width

I HAVE NO IDEA WHAT THE FUCK IS GOING ON. I HATE BEING LATE. FUCK! FUCK! FUCK!

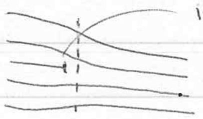
Defects / flaws

∘ Dislocation



$\tau_0 \approx \frac{G}{\pi}$ theoretical shear strength of crystal... too high

e.g. edge dislocation



insert an extra plane of atoms in

slip plane - atoms aren't locked down tight together

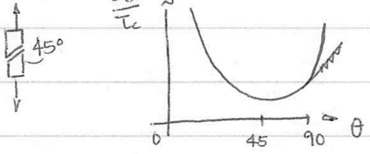
makes material easier to shear parallel to plane

$(\dashv)$

I HONESTLY FEEL LIKE I'M GOING (NO, WANT) TO DIE. WHY'D DIANE GO AND BREAK HER ARM? HONESTLY.

Necking

shear occurs – two planes sliding relative to each other

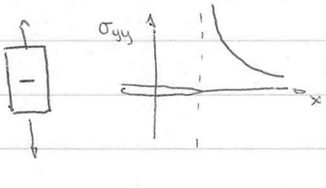


need shear stress parallel to slip plane

slip plane rotation

entropy helps govern dislocations

# of dislocations is never zero.



$$\boxed{\sigma_{yy} = \frac{\sigma_\infty}{\sqrt{\left(\frac{x}{a}\right)^2 - 1}}}$$

Stress at crack tip is singular

Hull & Bacon, <u>Introduction to Dislocations</u>, 1986 (Prentice Hall)

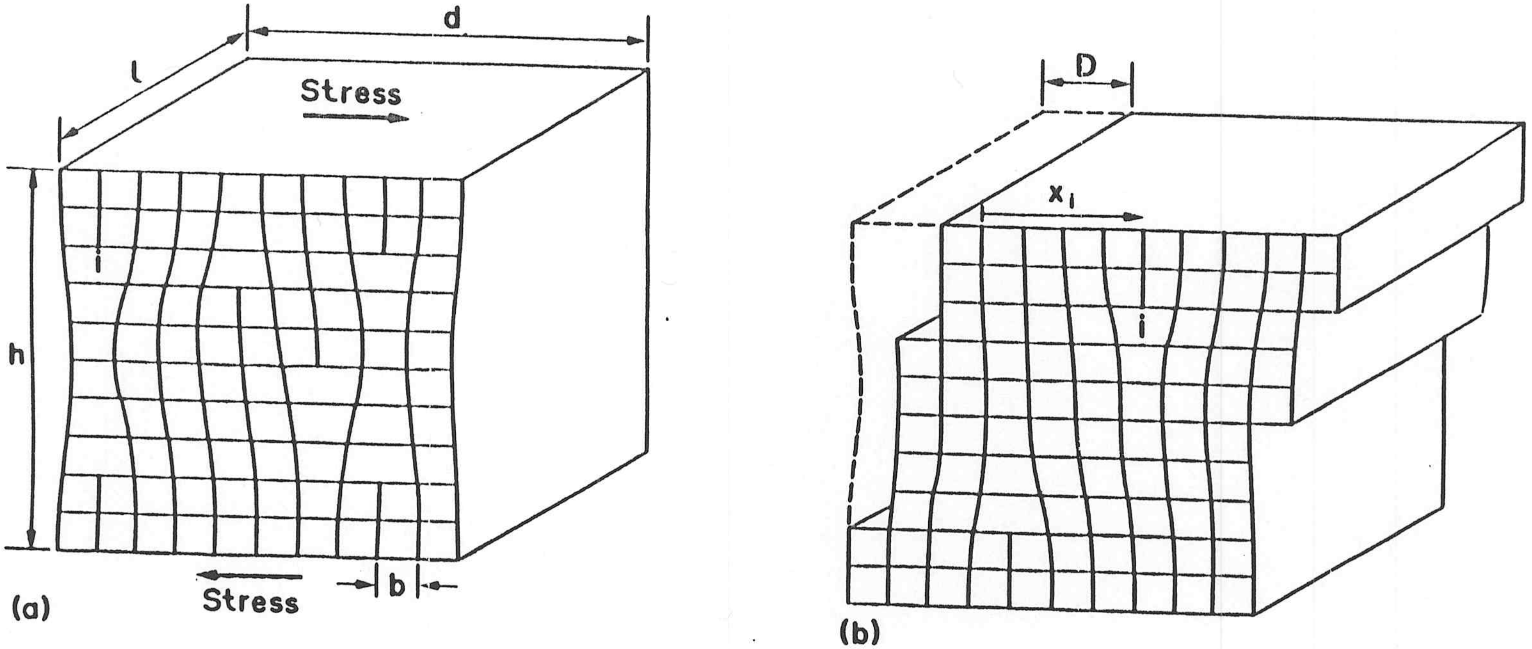


FIG. 3.22. (a) Edge dislocations in a crystal subjected to an external shear stress resolved for slip. (b) Plastic displacement D produced by glide of the dislocations. Dislocation i has moved a distance x_i, as shown.

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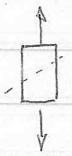
Wow I'm Behind

October 2, 2003

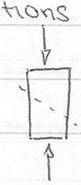
Response of materials

metals fail in shear by moving dislocations

metals = ~~d~~ ductile

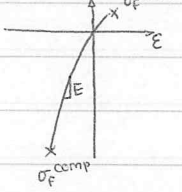


shears along 45° plane

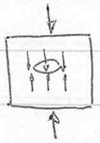


tension, compression behave in the same manner

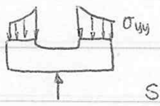
yield by dislocation movement



glass / ceramics in compression



crack gets pushed closed (?) yes.

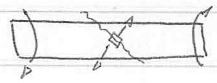


so how does the crack propagate in compression?

cracks don't open in compression parallel to existing cracks

crushing of brittle materials occurs

by the mechanism: cracking

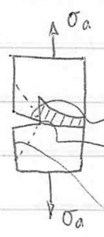


chalk example

Fracture mechanics

um. apparantly polymers act like metals

all cracks release energy proportional to $\underline{\text{const}} \cdot \frac{\sigma_a^2 \cdot a}{E}$



stress on crack surface = 0

σ_{yy}, highest at crack tip (∞)

not stressed much (inside dotted lines)

precrack: uniform stress

after: stress is small in triangle above / below crack, assume $\sigma = 0$

energy goes to:

- heat
- frictional sliding
- plastic dissipation

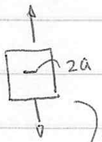
$\boxed{\Delta \text{ surface energy} = 2\gamma a \cdot t}$

a = crack length; t = thickness

γ = surface energy

strain energy in triangles before cracking

$\frac{1}{2}\sigma\varepsilon = \frac{E\text{strain}}{\text{vol}}$ S.E. = $2\gamma a t$ & $\boxed{\gamma = \frac{\sigma_a^2 a}{4E}}$



use W ≡ work of fracture, not γ → measured property, TOUGHNESS

$\boxed{\sigma_a = \left(\frac{4\gamma E}{a}\right)^{1/2}}$ critical stress for a given flaw size

$\boxed{a = \left(\frac{4\gamma E}{\sigma_a^2}\right)}$ critical flaw size for a given stress

$\boxed{\gamma = \frac{\pi \sigma_a^2 a}{E}}$

low estimate!

C.Hovell

<u>ARGH!</u> October 2,2003

$W \equiv$ work of fracture, or TOUGHNESS

example Ws for different materials:

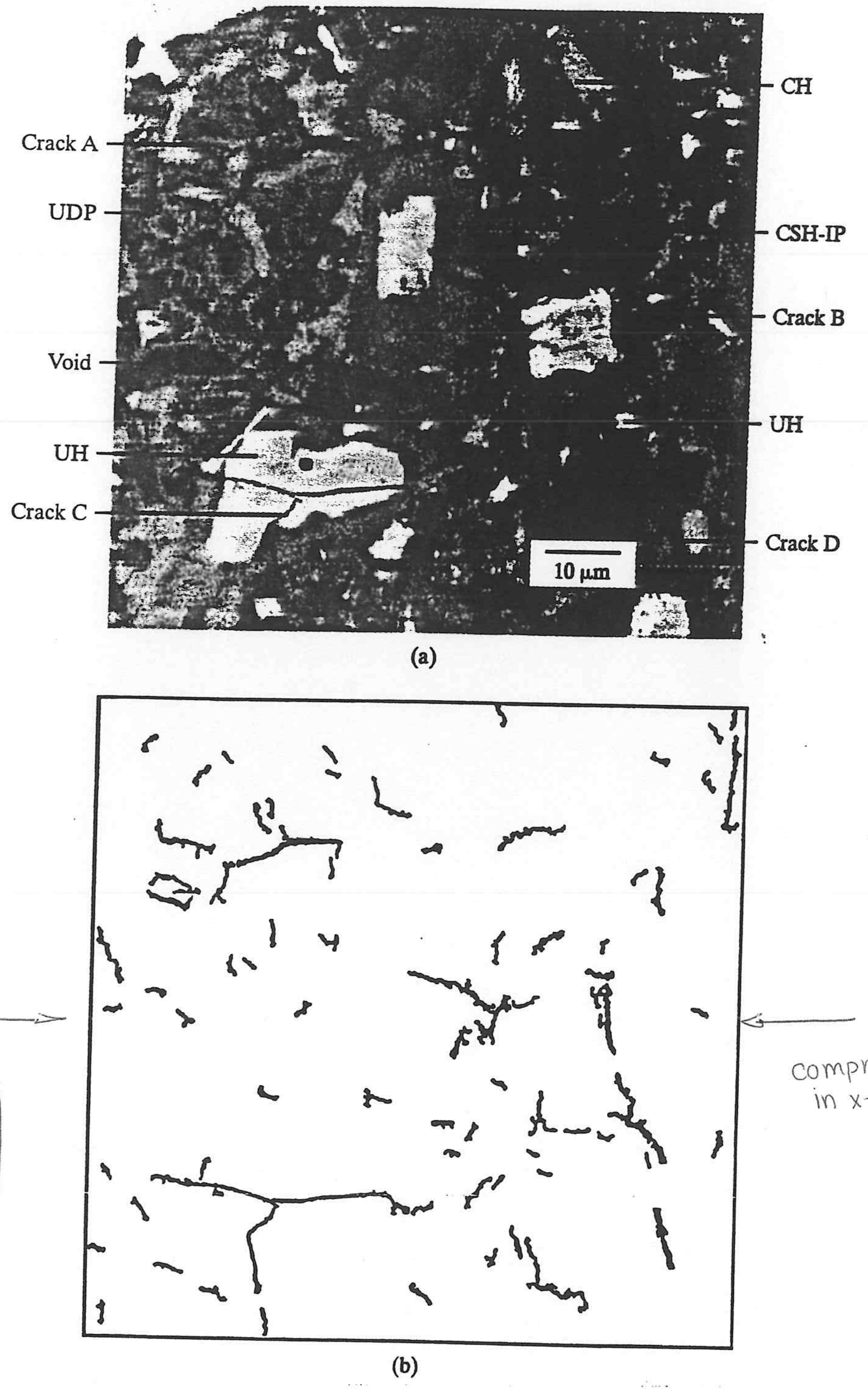
- glass/ceramics ~ 2-10 J/m^2
- wood ~ 10^4 J/m^2 (and manmade composites)
- metals ~ 10^6 J/m^2

metals are a million times tougher than ceramics

ARGH- I HATE THAT I HAD TO USE A SECOND SHEET. BAH HUMBUG. FUCKITY FUCK FUCK FUCKER.

metals are equal(ish) in strength to ceramics.

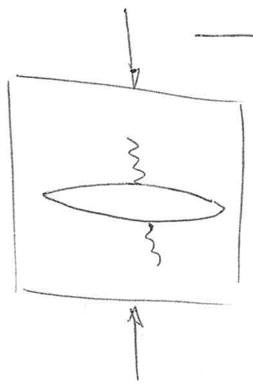
Concrete, Mindess, Young & Darwin, Prentice Hall, 20



(a)

(b)

compression in x-axis

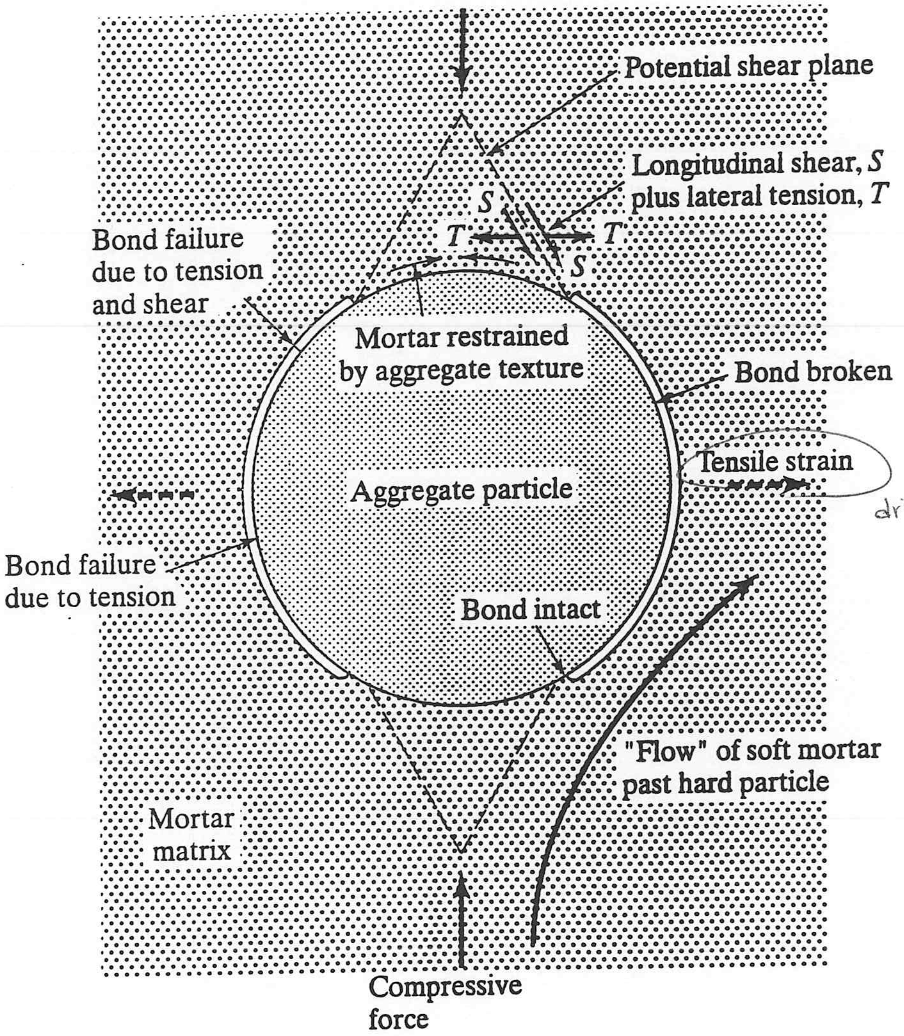


brittle objects fail by cracking parallel to comp. forces

FIGURE 13.23

Microcracking in bulk cement paste $w/c = 0.5$ loaded to a compressive strain of 0.006: (a) backscattered electron image (UH = unhydrated cement, CSH-IP = inner product); (b) crack map. (From D. Darwin, M. N. Abou-Zeid, and K. Ketcham, *Cement and Concrete Research*, Vol. 25, No. 3, pp. 605–616 (1995).]

Concrete, Mindess, Young & Darwin, Prentice Hall, 2002



drives cracks

FIGURE 13.25

Idealization of stresses around a single aggregate particle. [From G. W. D. Vile, *The Structure of Concrete*, ed. A.E. Brooks and K. Newman, Cement and Concrete Association, London, pp. 275–288 (1968). Reproduced by permission of British Cement Association, formerly Cement and Concrete Association.)

C.Hovell

My Life SUCKS

October ~~8~~ 7, 2003

Energy release rate

$G = \text{const} \cdot \frac{\sigma^2 a}{E}$ fracture happens when G hits a critical energy release rate

$G = G_c$

toughness - ability to withstand cracks

Griffith was strain energy —> surface energy

fracture at $G = \gamma$ $\gamma \equiv$ surface energy (does not work!)

however, strain energy goes to other energy losses, too.

G values:

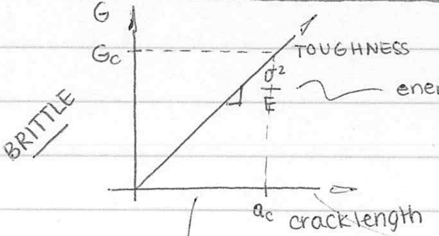
glass/ceramics ~ 2-10 J/m^2

wood ~ 10^4 J/m^2

metals ~ 10^6 J/m^2

Mechanical energy release rate

Fracture mechanics: for a given flaw and loading, compute G, the energy release rate — fracture at $G = G_c$ toughness



$\frac{\sigma^2}{E}$ ~ energy density (energy release rate)

a_c = critical crack length

→ energy released > toughness, material fails

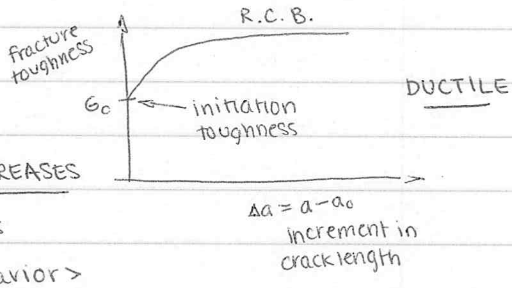
→ energy released is less than toughness, no effect

A little twist with real materials (like steel)

above graph great for glass

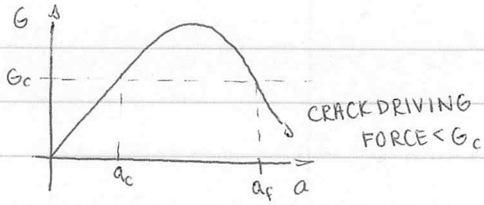
(toughness)
material resistance INCREASES as the crack grows

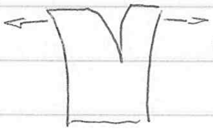
<resistance curve behavior>



going back to split log discussion...

wood does not show resistance curve behavior





better method :)

starts in tension, when it hits an area of compression, it stops

QUIZ #2 REVIEW

October 7, 2003

Crushing

compressive loading of brittle materials

crack propogation is VERTICAL due to tension

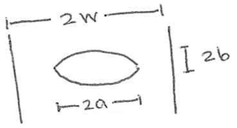
Material strength vs. specimen strength (component)

M.S. considers a perfect, unflawed version

no defects, all strengths computed on a scale of molecular separations

> intrinsic or theoretical strength

C.S. includes defects



$\sigma_{max} = \sigma_\infty (1 + 2^a/b)$

σ_0 = theoretical strength

σ_{max} = max stress in panel (needed to rip bonds)

$\sigma_0 = \sigma_\infty^{fail} (1 + 2^a/b)$

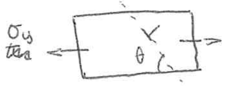
failure at $\sigma_0 = \sigma_{max}$

mechanism is bond rupture

if $b = 0$, $\sigma_{max} = \infty$, $\sigma_\infty^{fail} = 0$ ... switch to fracture mechanics method

Yielding

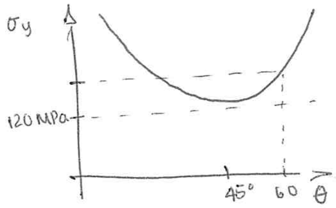
slip plane is the mechanism of yielding



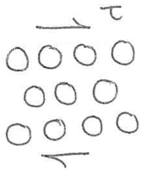
dislocation along slip plane at 60°

stress to move ⊥ is $\tau_c \approx 60$ MPa

what is σ_y?



dislocation may or may not be aligned with the best failure plane

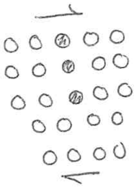


$\tau = \frac{G}{3 to 10}$

~20 GPa

theor. shear stress to shear a perfect crystal

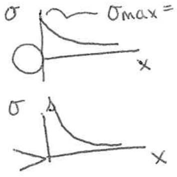
crazy huge!



critical shear stress to move ⊥

- use τ_c to consider dislocation movement because we'll ~~never~~ never have material without dislocations
- we can get better* results with σ_0 for tension because next to the ~~st~~ hole is approximately defect free

stress concentration vs. singularity



stress concentration

singular – close to ∞

$$\sigma_{yy} = \frac{c\sigma_\infty}{\sqrt{a^2 - x^2}}$$

if $x = a$, $\sigma_{yy} = \infty$

stress singularity is a lot worse

> fracture-based design

G = toughness

strength-based design σ_{max} = strength

CE323 Properties and Behavior of Materials
Fall 2003

Review for Quiz #2
Thursday, Oct. 9th

- Material selection: performance indices for:
    - bending stiffness for weight
    - buckling
    - strength for weight
    - energy storage
    - cost.

QUIZ #2 IS CUMULATIVE, SO KNOW THIS TOO!

- Strength of materials:
    - Hooke's law in three dimensions (with problems)
    - Elastic bending
    - Elastic-plastic bending

- Material properties:
    - Stiffness: elastic modulus
    - Strength: yield strength, ultimate tensile strength, fracture strength.
    - Toughness: *work of fracture, critical energy release rate*

Quiz 1

- Theoretical Strength of Materials:
    - Stretching of inter-atomic bonds
    - Conversion of strain energy to surface energy
    - Atomic slip in crystals: no dislocations

- Introduction to Defects and Failure Mechanisms:
    - Circular holes
    - Elliptical holes: component strength vs. intrinsic material strength (theoretical)
    - Stress concentrations vs. stress singularities
    - Dislocations: slip planes, component yield vs. critical shear stress
    - Crushing: compressive loading of brittle materials
    - Cracking: conversion of strain energy to surface energy, work of fracture, fracture toughness, critical energy release rate

---

Quiz 2

A few questions we've answered (in one way or another):

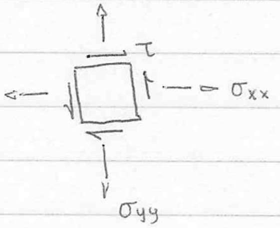
- What is the property ratio of density and strength that will optimize the performance of a component loaded in _____? (E.g. for torsion, maximize $\sigma_f^{2/3}/\rho$.)

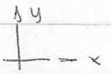
C.Hovell

Yay. Another 19.

October 16, 2003

Stress transformation

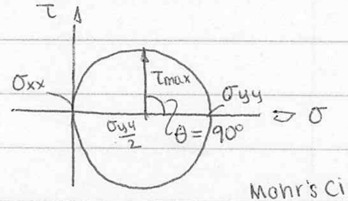




metals fail in shear

max shearstress occurs 45° from principal axes (where shearstress = 0)

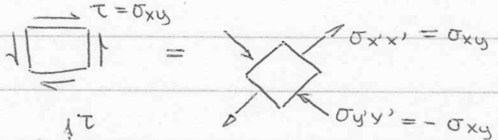
1. Pure direct stress

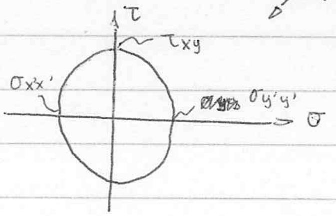


Mohr's circle has angles 2x real life

$\theta_{M.C} = 2\theta_{life}$

2. pure shear

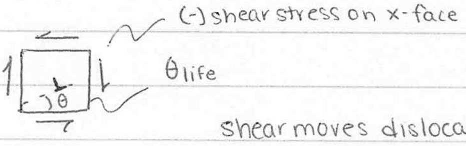




σ_{yy}, σ_{xx} at principal axes

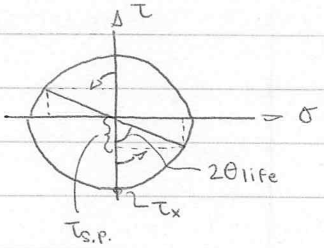
located 45° from given (pureshear) state

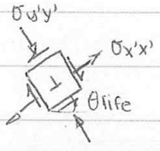
So, this semester:



shear moves dislocation along slip plane (θ_{life})

$\tau_{s.p.} = f$ (state of stress)





stress state acting on dislocation

only shear stress needed to move dislocation matters

$\tau_{s.p.} = \sigma_{xy} \cos 2\theta_{life}$

what if $\tau_{s.p} = \tau_{crit.} = \sigma_{xy}^{yield} \cos 2\theta_{l}$

$$\boxed{\tau_y = \frac{\tau_{crit}}{\cos 2\theta_{life}}}$$

C.Hovell

Rargh. October 21,2003

Failure theories (stress-based)

i. maximum normal (direct) stress criterion

for brittle materials

2. maximum shear stress

for dutile materials

criterion: tresca

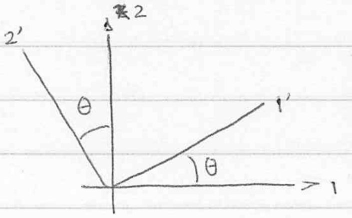
3. von mises criterion (or octahedral shear stress criterion)

So, how do we do this?

1. find σ_1, σ_2 at the principal axes

failure happens if $|\sigma_1, \sigma_2| > |\sigma_f^2, \sigma_f^t|$

Mohr's circle: 2-dimensional stress transformation



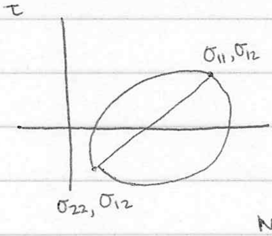
$$\boxed{\begin{aligned} \sigma_{11}' &= \sigma_{11}\cos^2\theta + 2\sigma_{12}\cos\theta\sin\theta + \sigma_{22}\sin^2\theta \\ \sigma_{22}' &= \sigma_{22}\cos^2\theta - 2\sigma_{12}\cos\theta\sin\theta + \sigma_{11}\sin^2\theta \\ \sigma_{12}' &= \sigma_{12}\cos 2\theta + (\sigma_{22}-\sigma_{11})\tfrac{1}{2}\sin 2\theta \end{aligned}}$$

where $\sigma_{12} = \tau$

$$\tan 2\theta_p = \frac{2\sigma_{12}}{\sigma_{11}-\sigma_{22}}$$

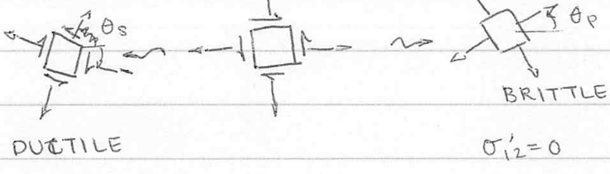
$$\tan 2\theta_s = \frac{-(\sigma_{11}-\sigma_{22})}{2\sigma_{12}} = \frac{-1}{\tan 2\theta_p}$$

$$\theta_s = |\theta_p \pm \pi/4|$$



$$R = \frac{\sigma_{11}-\sigma_{22}}{2}$$

Man I suck at drawing circles



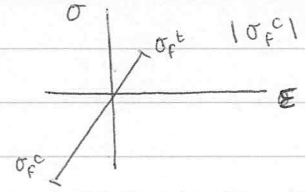
$\sigma_{11}' = \sigma_{22}'$; σ_{12}' max

$\sigma_{12}' = 0$

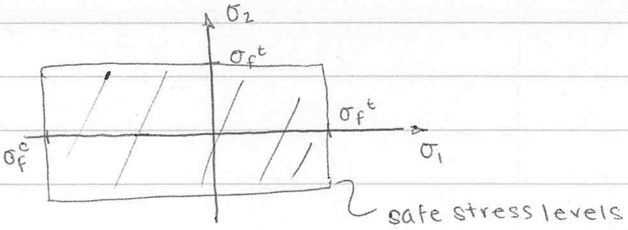
$\sigma_{11}', \sigma_{22}'$ max

Failure surface for a material

brittle – max normal stress



$|\sigma_f^c| > |\sigma_f^t|$



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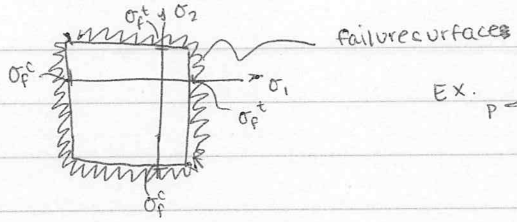
B00-Two pages

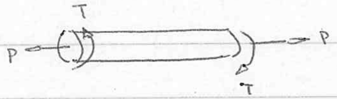
October 21, 2003

Failure surfaces

brittle materials (cont'd)



EX.



$\sigma_f^t = 200$ MPa

$\sigma_f^c = -700$ MPa

$P = 20$ kN $T = 600$ N·m $R_0 \sim 1$ cm

how does this fail?

- find principal stresses

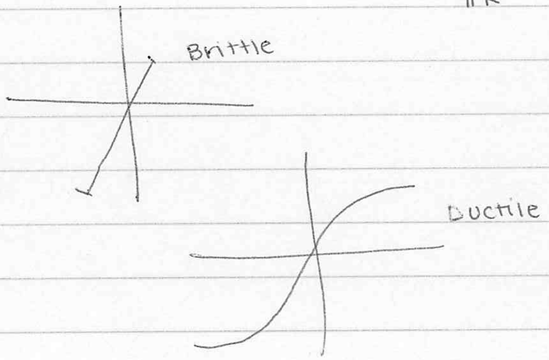
$$\tau = \frac{2T}{\pi R^3}$$

$\sigma_1 = 413$ MPa

$\sigma_2 = -350$ MPa

$\theta = 42°$

fails in tension



C. Hovell

7 Months to 21!

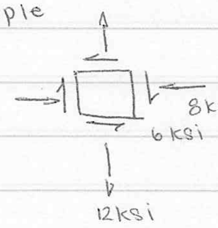
October 23, 2003

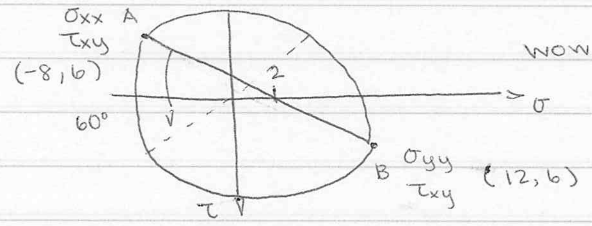
I DON'T GET WHY WE'RE GOING OVER MOHR'S CIRCLE ABOUT NINETEEN MILLION TIMES.

So, anyway...

if $\sigma_{yy} > \sigma_{xx}$, draw circle accordingly! (σ_{yy} to the right of σ_{xx})

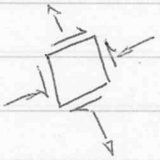
example



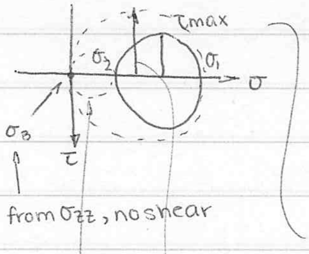


WOW. SUCKY-ASS CIRCLE

rotate 30°



Failure surfaces for ductile materials max shear stress (PLANE STRESS)



from σ_{zz}, no shear

→ τ_{max} in-plane shear stress

→ τ_{abs} or τ absolute max shear stress

no one cares about this dinky circle... yet.

HOLDS FOR ISOTROPIC MATERIALS

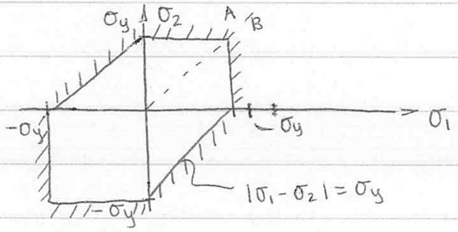
watch for σ_{zz}, which we tend to assume is zero - could be ≠ zero

do NOTHING with the extra circles except gather τ_{abs} - MOHR'S CIRCLE NOT POSSIBLE IN 3-DIMENSIONS!

max of $\frac{|\sigma_1 - \sigma_2|}{2}, \frac{|\sigma_1|}{2}, \frac{|\sigma_2|}{2}$

max is out of plane if σ_1, σ_2 are NOT on opposite sides of ~~τ=0~~ $\sigma = 0$

Failure (yield) surface



B will fail due to σ_1

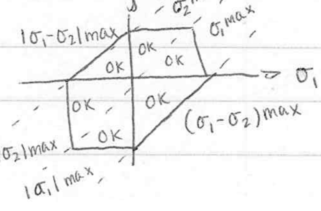
in quadrant I, III, $\frac{|\sigma_1|}{2}$ or $\frac{|\sigma_2|}{2}$ are max, cause failure

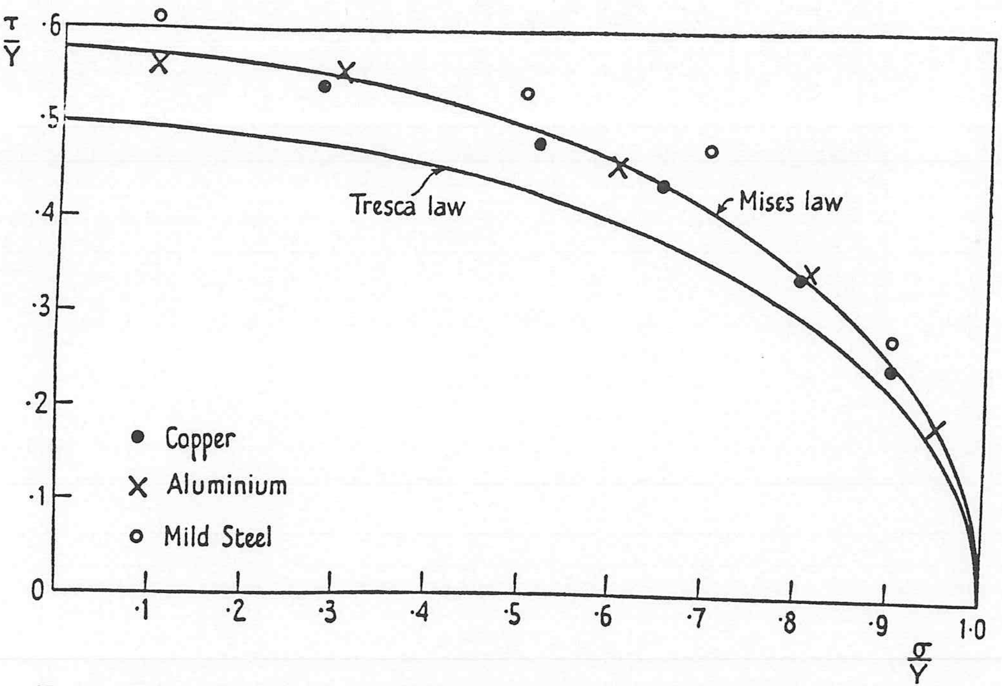
in quadrant II, IV, $\frac{|\sigma_1 - \sigma_2|}{2}$

yield surface: $\frac{|\sigma_1 - \sigma_2|}{2} = \pm \sigma_y$

$\sigma_1 = \pm \sigma_y \quad \sigma_2 = \pm \sigma_y$

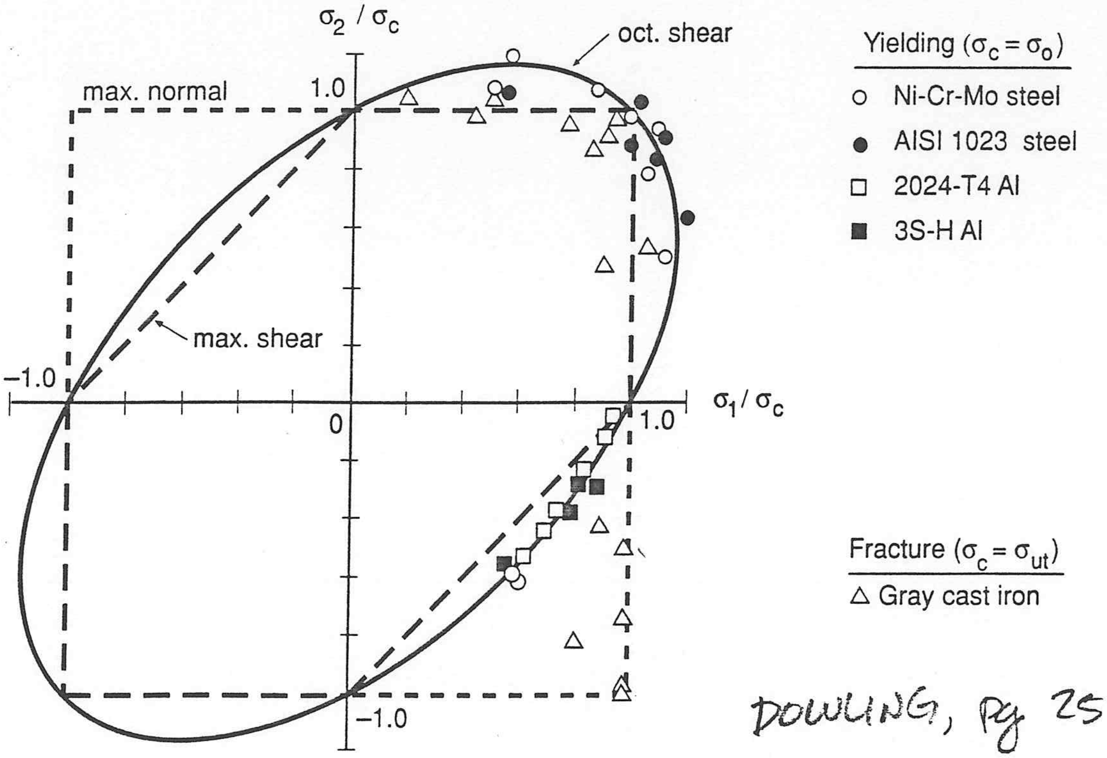
whichever is biggest





TAYLOR & QUINNEY'S EXPERIMENTS (1931)

FIG. 4. Experimental results of Taylor and Quinney from combined torsion and tension tests, each metal being work-hardened to the same state for all tests. The Mises law is $\sigma^2+3\tau^2 = Y^2$, while the Tresca law is $\sigma^2+4\tau^2 = Y^2$, where σ = tensile stress, τ = shear stress, Y = tensile yield stress.



DOWLING, pg 257

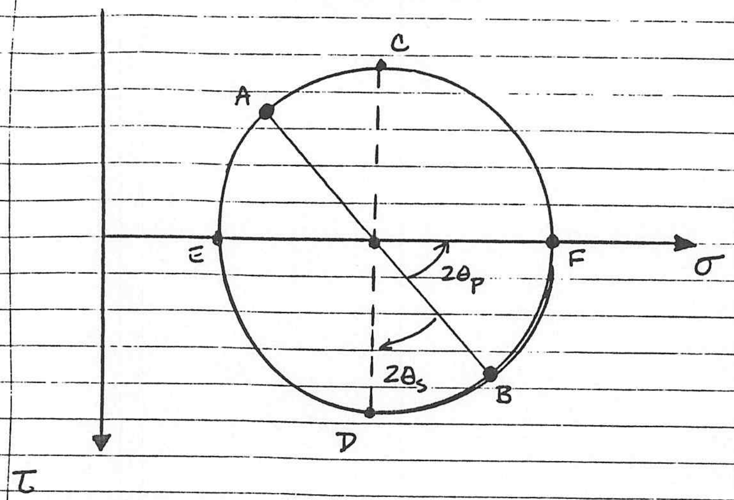
Figure 7.11 Plane stress failure loci for three criteria. These are compared with biaxial yield data for ductile steels and aluminum alloys, and also with biaxial fracture data for gray cast iron. (The steel data are from [Lessells 40] and [Davis 45], the aluminum data from [Naghdi 58] and [Marin 40], and the cast iron data from [Coffin 50] and [Grassi 49].)

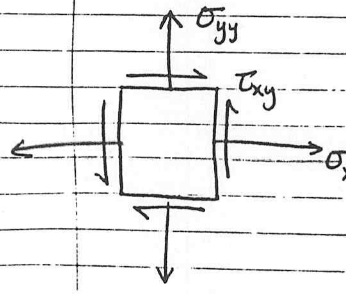
FIG-4 – R. HILL'S "MATHEMATICAL THEORY OF PLASTICITY", REPRINTED 1989

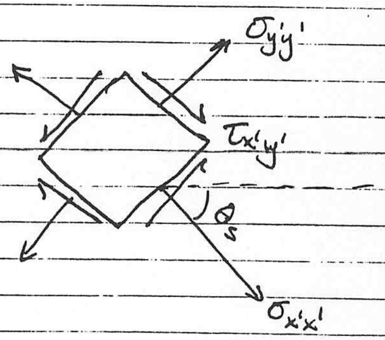
0|23|03

MOHR's CIRCLE

①







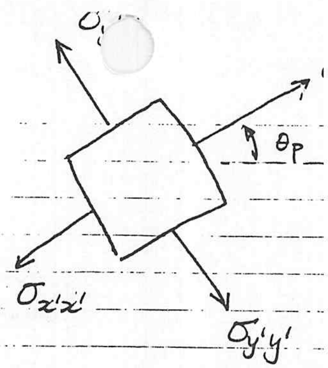
A-B: POSITIVE SHEAR ON X-FACE, NEG SHEAR ON y-FACE

POSITIVE (TENSION) ON BOTH x-y FACES.

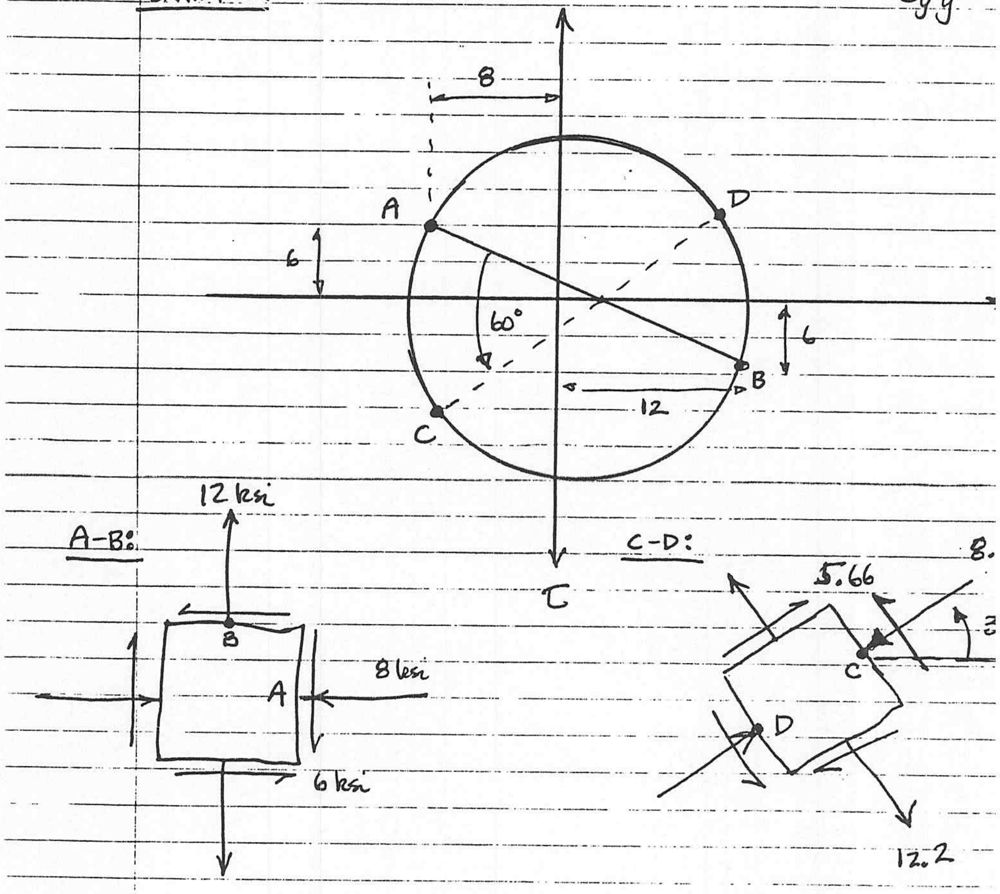
C-D: POSITIVE SHEAR ON x' FACE, NEG SHEAR ON y' FACE: TENSION ON BOTH FACES.

②

E-F: POSITIVE (TENSION) ON BOTH FACES: NO SHEAR



EXAMPLE:



A-B:

C-D:

C. Hovell

Man, My Nose Hurts

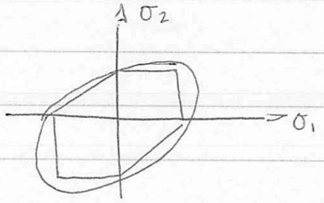
October 23, 2003

Failure surfaces for ductile materials

how can orientation of failure plane be determined?

we can't, unless $\tau_{max}^{abs} = \tau_{max}^{inplane}$ which only occurs when σ_1, σ_2 are of opposite signs

Von Mises criterion



$$\sigma_y = \frac{1}{\sqrt{2}}\sqrt{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2}$$

GOOD FOR ALL STRESS STATES!

for plane stress:

$$\boxed{\sigma_y = \frac{1}{\sqrt{2}}\sqrt{(\sigma_1-\sigma_2)^2+\sigma_1^2+\sigma_2^2}}$$

ellipse fits experimental data better than 

$$\sigma_y = \frac{1}{\sqrt{2}}\sqrt{(\sigma_{xx}-\sigma_{yy})^2+(\sigma_{yy}-\sigma_{zz})^2+(\sigma_{zz}-\sigma_{xx})^2+6(\sigma_{xy}^2+\sigma_{xz}^2+\sigma_{yz}^2)}$$

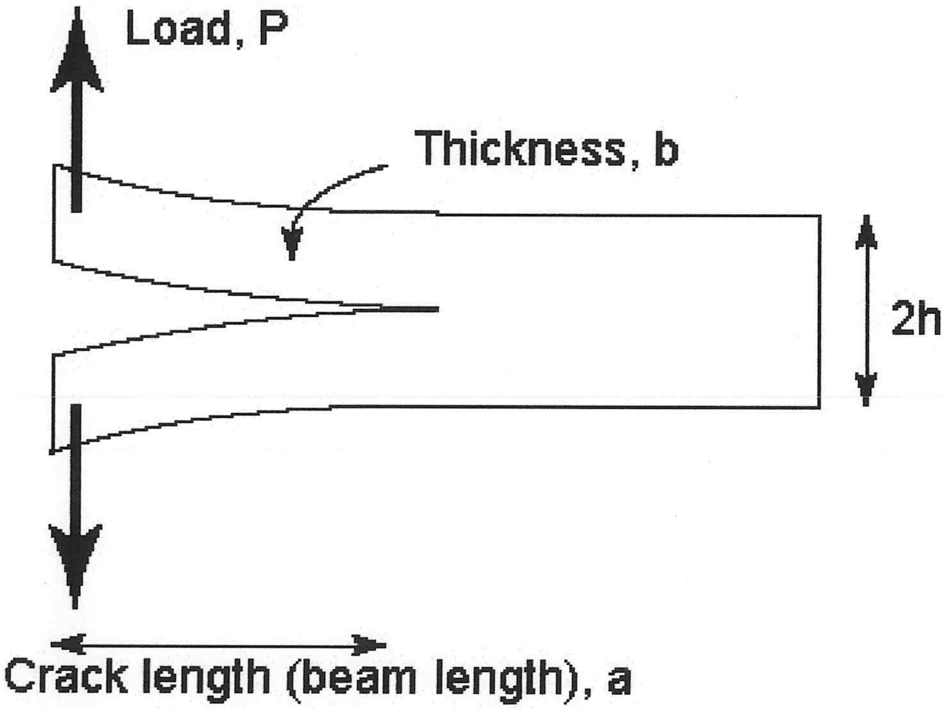
able to be used from any stress state, rather than from principal stresses

Invariant

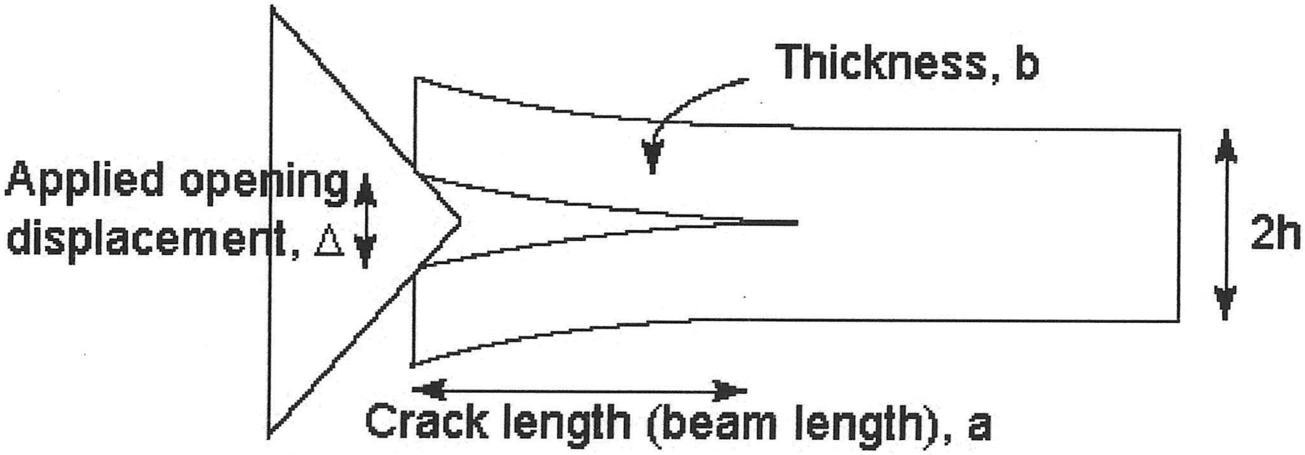
(to coordinate change)

does not matter how original piece is oriented, labelled

LOAD CONTROL



DISPLACEMENT CONTROL



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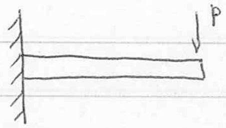
Test overview

October 30, 2003

Problem 1:

strongest: strength, failure stress, σ_f

lightest: density, ρ



$$\sigma_{max} = \frac{Mh}{2I} = \frac{Plh}{2I} = \frac{6Pl}{bh^2}$$

$$b = \left(\frac{6Pl}{\sigma_F}\right)^{1/2}$$

$$m = \rho h l b$$

$$m = \text{const} \frac{\rho}{\sigma_f^{1/2}}$$

minimize $\frac{\rho}{\sigma_f^{1/2}}$ or maximize $\frac{\sigma_f^{1/2}}{\rho}$

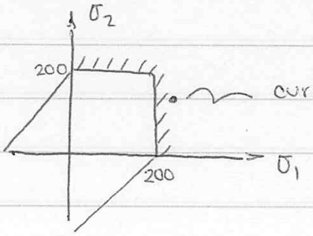
Problem 2: we'll skip this for now

Problem 3:

yield problem – use Mohr's circle to calculate principal stresses

(YAY! I GOT THE RIGHT NUMBERS, EVEN!)

$\tau_{abs} = \left|\frac{\sigma_1}{2}\right|$



yields according to max shear

$\frac{1}{2}\sigma_y = \tau_y$ and here, $\tau_{abs}^{max} = 110$ MPa,

$\tau_y = 100$ MPa

Von Mises:

$$\sigma_y \leq \sqrt{(\sigma_1 - \sigma_2)^2 + \sigma_1^2 + \sigma_2^2}$$

does not yield according to von Mises

→ matches the data better, but is less conservative

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Must wake up...

November 4, 2003

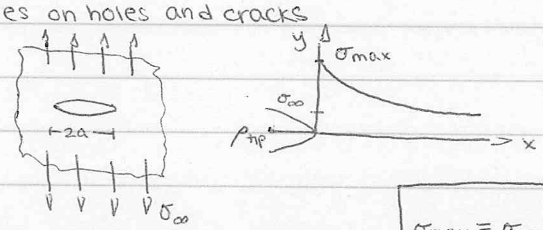
Toughness and fracture mechanics

- when do I use yield / failure theories and when do I use toughness / fracture theories?

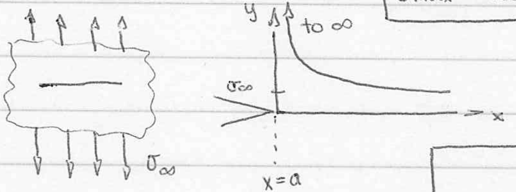
try both, which fails first?

Quick review

1. stresses on holes and cracks



$$\boxed{\sigma_{max} = \sigma_\infty \left(1 + 2\sqrt{\frac{a}{\rho}}\right)}$$



$$\boxed{\sigma_{yy} = \frac{\sigma_\infty a}{\sqrt{x^2 - a^2}}}$$

singularity at $x=a$, $\sigma_{yy}=\infty$

2. stress distributions around crack tips

stress at an angle θ from crack – consider $r \ll a$

for $\theta = 0$, $r \equiv x - a$

$$\boxed{\sigma_{yy} \cong \frac{\sqrt{a} \cdot \sigma_\infty}{\sqrt{2r}}}$$

square root singularity!

stress near the crack varies with the inverse of the square root of the distance from the crack

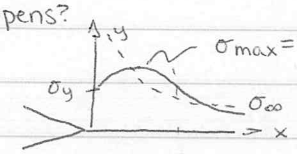
TRUE FOR EVERY CRACK TIP!

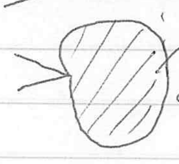
all stress distributions near crack tips of any geometry are the same!

magnitude of the stresses depend on the crack geometry and load (configuration)

what really happens?

Metals





plastic zone size is about $2r_p$

r_p = estimate

deforms plastically

outside plastic zone its elastic,

original approximation (model) works well

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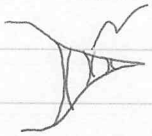
WOW. Half. Hour In...   November 4, 2003

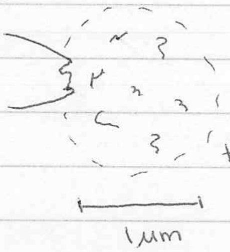
What really happens...

Polymers

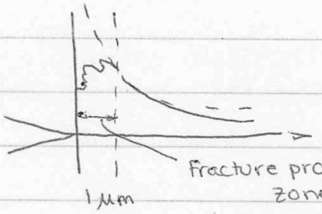


craze zone:
little ligaments (fibers ~~[illegible]~~) of polymer
bridging the crack

ceramics



microcracks appear ahead of
the main crack and lower stresses at the tip



component
↑
- small compared to us?
- large compared to crack length?

> if F.P.Z. is small compared to both criterion,
elastic analysis is fine

> small-scale yielding (SSY) ◁

Back to Review...

3. stress intensity factor

all crack tips have some
distribution of stress

$\sigma \sim \frac{1}{\sqrt{r}}$

more generally... $\sigma_{ij} = \frac{K}{\sqrt{2\pi r}} f_{ij}(\theta)$

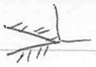
stresses are singular from any direction

> K ≡ stress intensity factor

function of geometry and loading

$K \equiv \lim_{r \to 0} \left(\sigma_{yy}(r) \cdot \sqrt{2\pi r} \right)$

$\boxed{K_I = \sigma_\infty \sqrt{\pi a}}$ parameter that characterizes/describes the crack tip



(for a center crack)

cracks happen when $K_I = K_{I\,crit}$

$\boxed{K_{Ic} \equiv \text{TOUGHNESS}}$

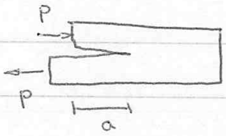
~~$\sigma_{fail} =$~~ $\boxed{\sigma_\infty^{fail} = \frac{K_{Ic}}{\sqrt{\pi a}}}$

$\sigma = \sqrt{\frac{E G_c}{\pi a}}$, or $G_c = \frac{K_{Ic}^2}{E}$

WTF-3 PAGES?!

November 4,2003

Aargh.



$$\frac{1}{A} = \frac{1}{A_1} + \frac{1}{A_2}$$

$$K_{II} = \frac{P}{\sqrt{2bA}}$$

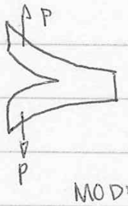
plane stress result

(K has a gazillion solutions for different geometries and loading types)

K_{I_c} = K-one-c (NOT $\underline{\underline{I}}$)

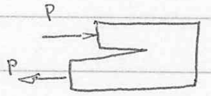
[NOTE TO SELF]

Failure / Fracture modes



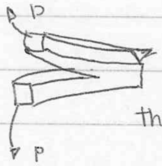
MODE I

opening mode



MODE II

Shear mode



think of tearing a piece of paper ☺

MODE III

tearing or anti-plane shear

TOUGHNESS (K_{Ic}, K_{IIc})

depends on mode!

we discuss Mode I most often, because usually,

~~$K_{Ic} < K_{IIc} < K_{IIIc}$~~ $K_{Ic} < K_{IIIc} < K_{IIc}$

opening usually has lowest toughness

LEFM ≡ linear elastic fracture mechanics

compute K for given geometry, loads via elastic analysis [look up!]

compare with K_{Ic} (toughness)

Units / values for K_{Ic}

$K_{Ic} \sim [\sigma][a]^{1/2}$ or $N/m^2 \cdot m^{1/2} = N/m^{3/2}$

really, MPa$\sqrt{m}$ or psi$\sqrt{in}$ of ksi$\sqrt{in}$

- metals 20-200 MPa$\sqrt{m}$
- polymers 1-5 MPa$\sqrt{m}$ → used at low stresses due to low yield strength
- ceramics 1-5 MPa$\sqrt{m}$

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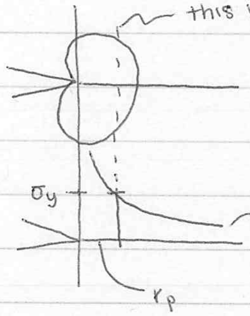
You're Joking, Right?

November 4, 2003

Plastic zone size



this line should be on edge ~~on~~ of bean

$\sigma_{yy} = \frac{K}{\sqrt{2\pi r}}$

$$\sigma_y = \frac{K_{Ic}}{\sqrt{2\pi r_p}} \qquad \boxed{r_p \sim \frac{1}{2\pi}\left(\frac{K_{Ic}}{\sigma_y}\right)^2}$$

In reality, the actual yield zone size is about twice as big

$$r_p^{act} \sim \frac{1}{\pi}\left(\frac{K_{Ic}}{\sigma_y}\right)^2$$

$$r_p < \frac{\text{smallest dimension}}{10}$$

C. Hovell

I wanna Be A Bunny

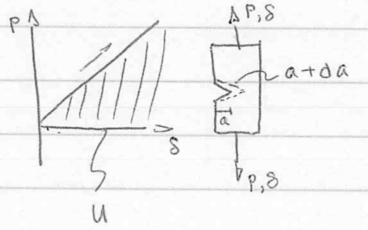
November 6, 2003

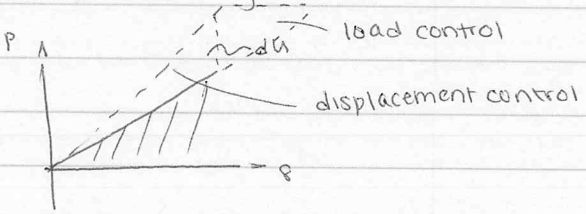
Why do we care about the plastic zone?

if it is too large, LEFM is not a good plan

(strain) Energy Release Rate (ERR)

> mechanical (strain) energy released due to the advancing of a crack





$$G = -\frac{1}{b}\frac{dU}{da}$$

> fracture happens when $G \geq G_c$ (critical ERR) or TOUGHNESS (J/m^2)

the amount of energy needed to create new crack surfaces

* > Irwin Relationship

$$G = \frac{K^2}{E} \text{ planestress} \qquad G = \frac{(1-\nu^2)K^2}{E} \text{ planestrain}$$

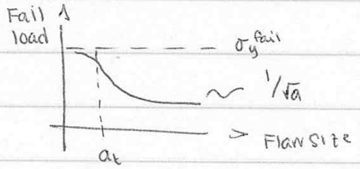
$$\boxed{G_c = \frac{K_{Ic}^2}{E} + \frac{K_{IIc}^2}{E_*} + \frac{K_{IIIc}^2}{E_*}}$$

$$K = \sqrt{2\pi r\, \sigma_{yy}^2}$$

metals $G_c \sim 1\times10^3 \to 1\times10^5$ Pa·m

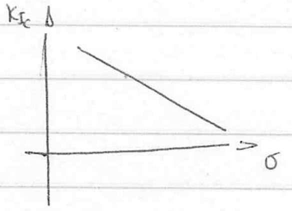
$\text{Pam} = \cancel{\text{Pa } J^2}\ J/m^2$

Shit! Falling asleep is BAD.



$K = \sigma_\infty^{fail}\sqrt{\pi a} = K_{Ic}$

$\sigma_{fail} = \frac{K_{Ic}}{\sqrt{\pi a}}$



a_t = transistion flaw size

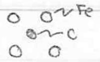
as strength increases, toughness decreases

must find optimum combination of K_{Ic}, σ_y

Toughness mechanisms

> alloys are full of crap.

(Nevermind solid solutions



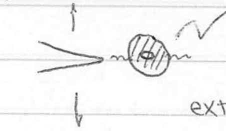
substitutional solid solution

interstitial solid solution

DON'T AFFECT TOUGHNESS SO MUCH.

◦ precipitates

Brittle Failure



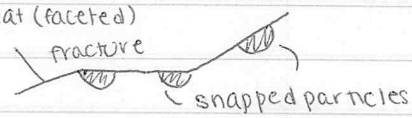
"weak" relative to the matrix

rapid failure of particle can

extend to the matrix on both sides

CLEAVAGE FAILURE

Flat (faceted) fracture



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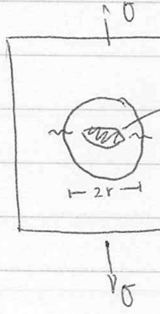
And with 10 Minutes Left...

November 6, 2003

Brittle Failure, still



flaw in cracked particle

penny shaped flaw: $K = \frac{2}{\sqrt{\pi}} \sigma \sqrt{R}$

(stress intensity factor of crack extending into matrix)

- strength of the particle scales with

$\sim 1/\sqrt{R}$

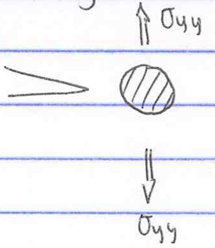
smaller particles are good (for this mechanism)

avoid cleavage failures by having small particles

I Hate New Paper

November 12, 2003

Toughening Mechanisms: particle effects



- particle breaks (before the crack reaches it)
- crack zips into ductile materials
- cleavage fracture

the likelihood of particle failure goes up as the particle size goes up

$\boxed{G_c^{\text{overall}} \alpha {}^1/R^{1/2}}$ $a = R$

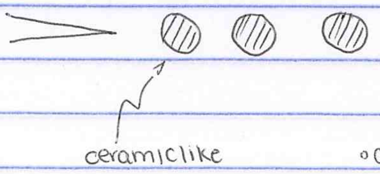
particle introduces a microcrack ahead of the dominant crack

K (stress intensity factor) scales $\sim 1/\sqrt{R}$

crack extends when

$K_{\text{microcrack}} = K_c$ of matrix near particle

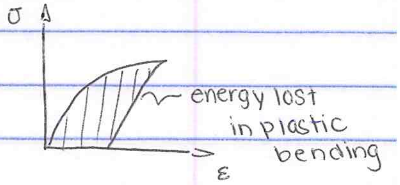
Ductile Toughening Mechanism



- flaws make the material more tough!
- particles affect local stress field

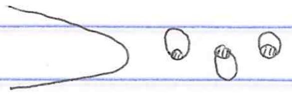
increases in stress increase plastic flow, so

- anything that increases plastic flow makes it harder to drive the crack and increases toughness



- strength for one material may be high, but energy is dissipated by low strength / ductile material

strong or tough, not both



→ particles debond (separate) from the matrix

→ microvoids form

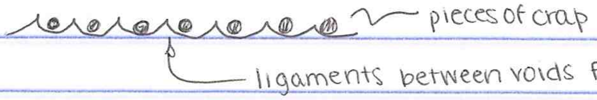
→ voids grow into one another

$\boxed{\text{MICRO-VOID COALESCENCE}}$

ductile fracture mechanism

(particle rupture is okay, so long as crack stops at matrix (doesn't usually work))

Fracture surface



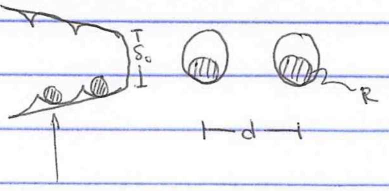
pieces of crap

ligaments between voids fail by tension through necking

I'M STILL AWAKE.

November 11, 2003

Ductile Toughening



δ_c = opening of the blunted crack

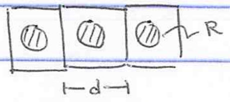
d = particle spacing

R = radius of crap

hills, valleys from previous pieces of crap

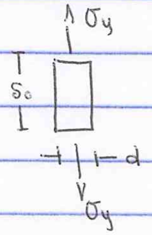
$$\boxed{\frac{R}{d} \sim f^{1/3}}$$

f ≡ volume fraction of particles (ratio of particle volume to matrix volume)

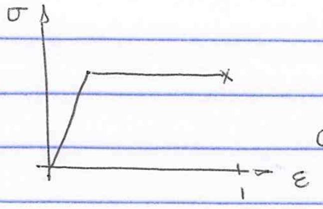


$$\frac{R^3}{d^3} \sim f$$

what about between particles?



$\varepsilon_f \sim 1$



energy dissapated = area under the curve

work done on ligament ~ $\sigma_y \delta_o$

crack opening at ligament failure $\delta_o \sim d$

Estimate of Toughness K_c

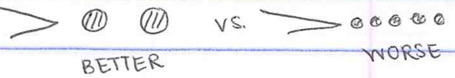
$$\boxed{\frac{K_c^2}{E} = G_c \sim \sigma_y d}$$

or

$$\boxed{K_c \cong \sqrt{\frac{E \sigma_y R}{f^{1/3}}}}$$

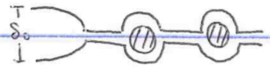
for the same amount of particles by volume, bigger particles are better, as they make the DUCTILE material tougher

- particles, if too big, cause brittle failure mechanism by failing quickly and easily.



- there is some optimal particle size to be found and utilized
- you can ~~inadvertently~~ inadvertently switch mechanisms while trying to enhance your favorite!

High strength:



strength and toughness are usually inversely related – strong materials aren't tough!

ligament fails before $\delta_o \sim \varepsilon_o$

This Might Be Interesting

November 13, 2003

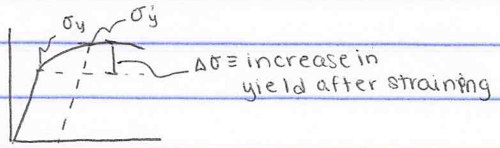
Strengthening Methods / Mechanisms for METALS: → alloys

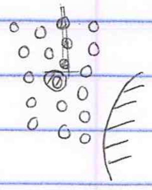
altering the way the material wants to deform

> high yield stress = strong

- stop dislocations
- create obstacles to dislocation motion
  - create immobile (sessile) dislocation tangles

strain hardening





  - substitute impurity atoms : solid solutions

replace small atom with C, Fe to cause a blockage

  - particles: precipitates and dispersoids
  - intrinsic lattice resistance (controlled by bonding)

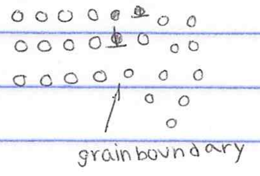
$\tau_b \equiv$ Peierl's stress (peierl's – Nabarro)

previously called $\tau_c \equiv$ critical shear stress to move ⊥

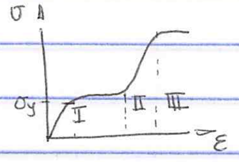
  - grains: individual crystals

to stop dislocations, change grain size

grain boundaries are the obstacle to dislocation movement



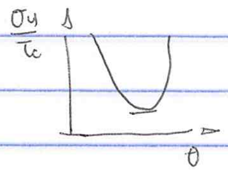
Yielding in single crystals (e.g. individual grain)

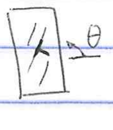


initial ~~yied~~ yield stress

$\sigma_y^i = \tau_p / \text{fcn of orientation}$

$\boxed{\sigma_y^i = (2\text{-}3)\,\tau_p}$





> Region I: stage hardening

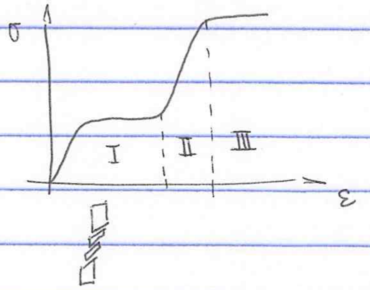
one slip ~~plane~~ system operates

very little ⊥ ~~ab~~ interaction

"single slip"

Must... Stay... Awake. November 13, 2003

Yielding in single crystals



$\frac{d\sigma}{d\varepsilon} = \sim (10^{-3} \text{ to } 10^{-4}) G$ very small – no strain hardening

Shear yield stress of material

$\tau_I = \tau_p + \alpha_I G b \sqrt{\rho}$

shear yield in stage I

lattice resistance

a constant

shear modulus

As $b\uparrow, \tau_y\uparrow$ } can't

$G\uparrow, \tau_y\uparrow$ } change

As $\rho\uparrow, \tau_y\uparrow$ – ρ can be controlled

b = Berger's vector, "size" of a dislocation

ρ = density of dislocations

how to change dislocation density?

bang it with a hammer

common values

$\alpha_I \sim 1/20$ annealed: $\rho \sim 10^7 \perp/cm^2$ (they're very small!)

worked: $\rho \sim 10^{11} \perp/cm^2$

$\Delta\tau \equiv$ change in yield due to $\perp$ movement $\sim$ 0.1 to 10 MPa

(not a big number compared to σ_y)

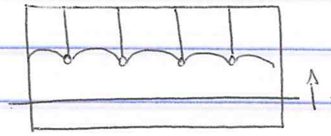
Stage II Hardening

multiple slip on multiple slip systems – cross slip

- $\perp$ interaction goes through the roof (becomes complicated)
- leads to enormous increases in strength, as interactions lock the $\perp$

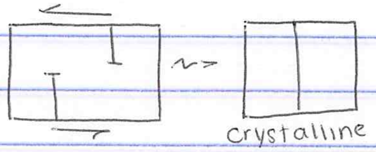
$\tau_{II} = \tau_p + \alpha_{II} G b \sqrt{\rho}$

$\alpha_{II} \sim 0.2$ to 0.4



$\sim$ lower dislocation bows between obstacles, gets caught on edge of other dislocations

Stage III Hardening



$\rightarrow$ dislocation annihilation leads to dynamic recovery

ρ (density) saturates, or decreases

Dislocation annihilation – certain types of dislocations eliminate each other

Woah-One Lab Back

November 18, 2003

Strengthening Mechanisms

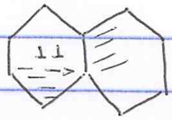
- intrinsic lattice resistence, τ_c (bonding)
- 2^{nd} phase particles
- impurity atoms
- immobilize $\perp$ with tangles
- grain boundaries

yield in polycrystals

only stage II (cross-slip) and stage III (dislocation annihilation) occur

no stage I because slip happens immediately on multiple slip systems

grain size effect

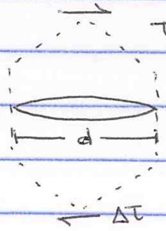


upon hitting grain boundary...

$\perp$ stops, nardening occurs - "dislocation pileup"

little crystals are better, stronger

treat $\perp$s on slip plane like a crack



$\tau_a - \tau_p = \Delta\tau$

τ_p = intrinsic strength of material

Mode II

$$\boxed{K_{II} = \Delta\tau\sqrt{\frac{\pi d}{2}} = K_{II_c}}$$

$$\Delta\tau = \frac{K_{IIc}}{\sqrt{\pi/2}\sqrt{d}}$$

↳ relates to the strength of the boundary

$$\tau_y = \tau_p + \frac{C_0}{\sqrt{d}}$$

d = grain size

C_0 = constant

τ_y = shear yield stress

τ_p = lattice resistence - shear stress to move $\perp$

how does strength vary with grain size?

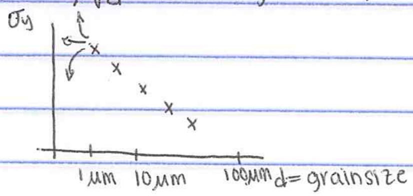
zero grain size, infinite yield resistance

Hall-Petch Relationship

$$\sigma_y = \sigma_0 + k_y/\sqrt{d}$$

k_y is in $N/m^{3/2}$

VERY VERY IMPORTANT!



100 × 100 atoms = 300 nm = smallest crystal (ish)

nanograined materials: d ~ 10-100 nm, smallest

And so it continues...

November 18, 2003

Hall-Petch Relationship - still really important

grain sizes adjusted via annealing

grains grow at elevated temperatures

quench heated metal - throw cold, cold water in

sample numbers...

face-centered cubic Al $k_y = 0.07\ MN/m^{3/2}$

Cu $k_y = 0.10\ MN/m^{3/2}$

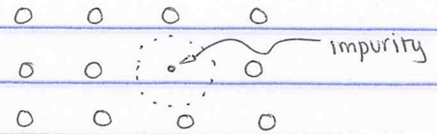
body-centered cubic Fe $k_y \sim 0.3$

steel $k_y \sim 0.3$

when $d \sim 1\ \mu m$, $\Delta\sigma \sim 300$ MPa, which is much higher than τ_p

Solid Solution Strengthening

> substitutional



to increase strength, impurity must stop dislocations.

$R_i \neq R_h$ (atoms must be of a different size, radius)

lattice distortion -

interacts with dislocation to increase resistance to plastic flow

$$\boxed{\Delta\sigma = \beta(\varepsilon_*)\sqrt{c}}$$
$\cdot G$

$\varepsilon_* = \frac{R_i - R_h}{R_h} << 1$ dilation strain

$\beta \equiv$ constant that depends on $\varepsilon_* \sim 0.0005 - 0.005$

$\Delta\sigma$ = increase in σ_y over pure material

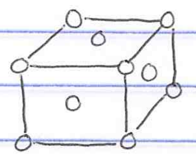
G = shear modulus

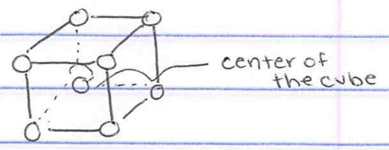
c = concentration

Structures

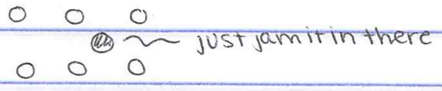
FCC = Face-centered cubic

BCC = body-centered cubic





> Interstitial

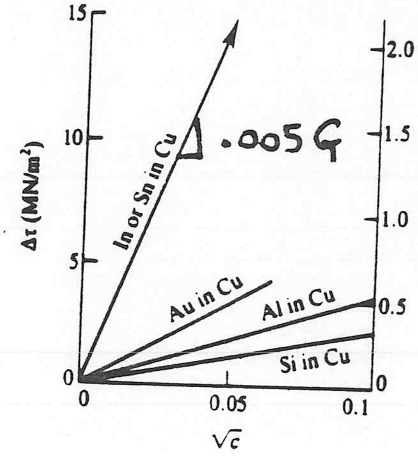


$\beta(\varepsilon_*) \sim 0.005 - 0.01$ for FCC

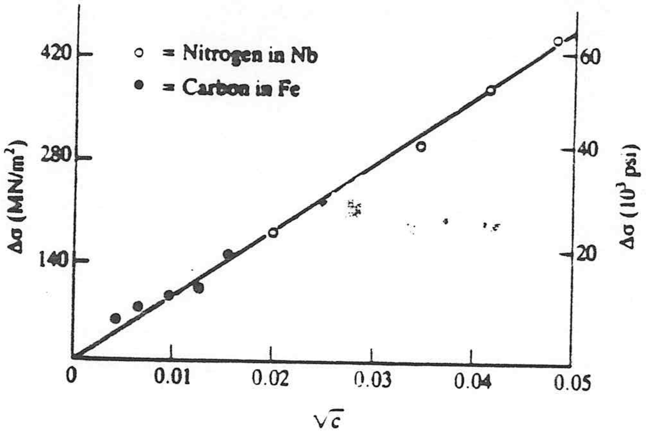
$\beta(\varepsilon_*) \sim 0.05 - 0.1$ for BCC

EXAMPLES OF SOLID SOLUTION STRENGTHENING

(a) SUBSTITUTIONAL



(b) INTERSTITIAL



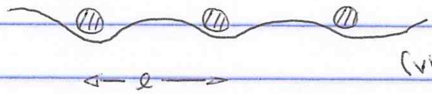
Test in 30 Minutes

November 18, 2003

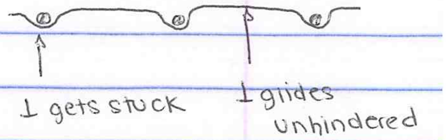
Precipitate / Dispersoids effect on yields

dislocation front bows

Possibility A:



(view from the top)



$$\boxed{\ell = \frac{R}{\sqrt{f}}}$$

f ≡ volume fraction

R ≡ radius of particles

stress needed to bow a dislocation

$\tau_{bow} \propto Gb\sqrt{f/R}$ ~ basically, put lots of small particles close together, ~~ratrra~~ rather than big particles spread apart

Possibility B:

particle can fail through Mode II (shear)

antiphase boundary

dislocation continues, prolly with a new extra half-plane

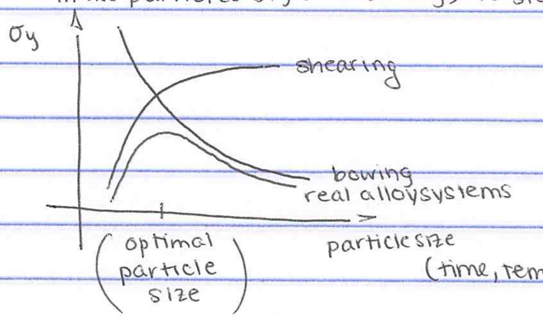
energy needed to shear particle = $\pi R^2 \gamma_{APB}$

APB = antiphase boundary

$$\tau_{shear} \propto (\gamma_{APB})^{3/2} \left(\frac{R \cdot f}{b}\right)^{1/2}$$

b ≡ constant, length scale

make particles big (and strong) to stop dislocations



Big
- good for unshearable particles $\sigma_y \uparrow$
- bad for bowing $\perp$s $\sigma_y \rightarrow$
- bad for brittle fracture $K_{Ic} \downarrow$
- good for ductile fracture $K_{Ic} \uparrow$

Small
- good for bowing $\sigma_y \uparrow$
- bad for shearing $\sigma_y \rightarrow$
- good for brittle fracture $K_{Ic} \uparrow$
- bad for ductile fracture $K_{Ic} \rightarrow$

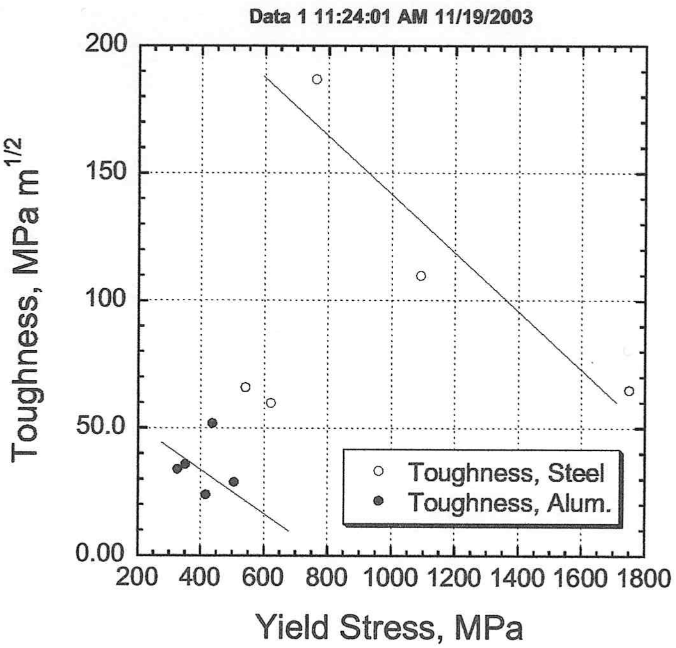
MAN I HATE TAKING THIS MUCH NOTES.

CE323 Properties and Behavior of Materials **Homework #5**
Fall 2003 **Due: Tuesday, Oct. 18th**

SOLUTIONS

Material	Yield stress, σ_y (MPa)	Toughness, K_{Ic} (MPa $m^{1/2}$)	Transition flaw size (mm)	Plastic zone size (mm)
AISI 1144 (steel)	540	66	4.76	2.38
ASTM A470-8 (Cr-Mo-V)	620	60	2.98	1.49
ASTM A517-F	760	187	19.3	9.64
AISI 4130	1090	110	3.24	1.62
300-M (300oC temper)	1750	65	0.44	0.22
2014-T651	415	24	1.06	0.53
2024-T351	325	34	3.48	1.74
2219-T851	350	36	3.37	1.68
7075-T651	505	29	1.05	0.52
7475-T7351	435	52	4.55	2.27

Problem One:



Very generally speaking, a material becomes less tough as the yield strength increases. Increases in yield strength mean that it is more difficult to move dislocations; this means that there is less plastic flow (or deformation) near crack tips. Plastic flow near crack tips is one mechanism that materials use to dissipate energy; this increases toughness by converting elastic strain energy to plastic work, instead of crack growth. Another way to look at it is that materials with high strength remain primarily elastic, storing more elastic strain energy to advance a crack.

The plot does not reveal a very obvious trend because there aren't enough data points for different materials. For a given alloy, the trend might be eliminated due to changes in the fracture mechanism: for example, if the size of hard particles is changed for a given steel, you might significantly change the toughness, without affecting the yield stress too much. A more consistent data set that illustrates the global trends is provided by Ashby, who has a materials selection map involving K_{Ic} and σ_y. This map reveals that materials that are less ductile (i.e. high strength, low failure strain) generally have smaller toughness.

Problem Two:

The transition flaw size is the size at which the failure stress predicted via linear elastic fracture mechanics (LEFM) is equal to the yield stress. This is determined by:

$$\sigma_f^{LEFM} = \frac{K_{Ic}}{\sqrt{\pi a_t}} = \sigma_y \rightarrow a_t = \frac{1}{\pi}\left(\frac{K_{Ic}}{\sigma_y}\right)^2$$

For flaws near the transition, fracture and generally yielding are equally likely. For flaws less than the transition flaw size, the failure stress predicted by LEFM is larger than the yield stress; hence, as the sample is loaded, the entire plate will yield before the crack grows. Conversely, for flaws greater than the transition flaw size, the fracture stress is less than the yield stress: this implies the crack grows before the plate undergoes general yielding. Values for the transition flaw size are included in the table above.

In specific cases, either is acceptable. The key consideration with regards to flaw size is that you want the transition flaw size to be big enough to detect. If the critical flaw size is too small, you'll never detect it in service and failure may be the first indication you have a crack. Generally speaking, the consequences of aerospace component failure are more dire, such that undetected but dangerous flaws are to be avoided at all costs; hence, choose a material with as large a transition flaw size as possible. Conversely, civil structures are typically easier to inspect (their infrastructure being fairly open) and more conservatively designed (weight not being a problem), so they are typically (but not always) strength-based design.) This might seem like a bit of hand-waving – it is, as the original question is poorly worded: in the future, we'll deal with specific case studies that have more details to consider.

Which theory to use? If you find a flaw larger than the transition flaw size, use LEFM. If you do not, ***and*** your detection system is sensitive enough to find flaws bigger than the transition flaw size, you can predict failure via plastic yielding. A very dangerous scenario: your transition flaw size is 0.1 mm, but your detection procedure (e.g. looking at it), can only detect flaws > 1 mm. What to do? Assume the worst – i.e. there is a flaw 0.99 mm, and predict failure based on LEFM.

Problem Three:

The stress distribution directly ahead of a crack tip is given by:

$$\sigma_{yy}(r) = \frac{K}{\sqrt{2\pi r}}$$

where it should be noted that $f_{ij}(\theta = 0) = 1$. The stress intensity factor for a crack in a panel that is much, much wider than the crack length (equal to $2a$) is:

$$K = \sigma\sqrt{\pi a}$$

where σ is the stress applied to the panel. If we assume that the materials at all locations where $\sigma_{yy} > \sigma_Y$, then the plastic zone size for a given applied stress level is:

$$r_p \approx \frac{a}{2}\left(\frac{\sigma}{\sigma_Y}\right)^2$$

The maximum plastic zone size is generated at the highest possible load: i.e., when the plate fractures, in which case: $\sigma \to K_{Ic} / \sqrt{\pi a}$. Thus, the plastic zone size at failure can be estimated as:

$$r_p^{\max} \approx \frac{1}{2\pi}\left(\frac{K_{Ic}}{\sigma_Y}\right)^2$$

The estimated plastic zone size at failure is shown in the original table. Brittle materials are easiest to determine the fracture toughness for, ductile materials are the hardest. Why? Because brittle materials have little plastic flow, and LEFM can be used for pretty much any sized specimen. Conversely, for materials that undergo lots of plasticity, a large specimen is required to ensure small scale yielding (SSY), which means the size of the plastic zone (or fracture process zone) is small compared to other length scales. If the region of plasticity is small, it can be ignored and LEFM can be used. For 300-M (300oC temper) steel, the strength is high, the toughness is low, and plastic zone size is about twice the size of a human hair: in this case, you can choose small specimens that will fail and small loads; this is the easiest to test from the above material set.

So, the specimen dimensions have to be chosen such that LEFM can be applied, i.e. such that the plastic zone size is small relative to all other dimensions. Ductile and tough materials generate large plastic zones before fracture (see above), so that very large specimens are required. This implies the applied loads get very large, since the load required to generate the failure stress scale with the cross section area of the specimen.

The hardest material to measure K_{Ic} for is ASTM A517-F. (Note: ASTM = American Society for Testing and Materials.). Why? Consider the estimated plastic zone size of ~ 2 cm. This implies that the crack introduced to measure toughness must be much larger, say:

$$a > 10r_p^{\max} \approx 20\,\text{cm}.$$

But to use the solution for a center crack in a large panel, the crack should be considerably smaller than the panel width, so that:

$$W > 10a > 100r_p^{\max} \approx 200\,\text{cm}.$$

This is a *big* panel, approximately 6'x6'. Suppose the steel was ¼" = 6 mm thick. The failure load for a crack and panel of these dimension is:

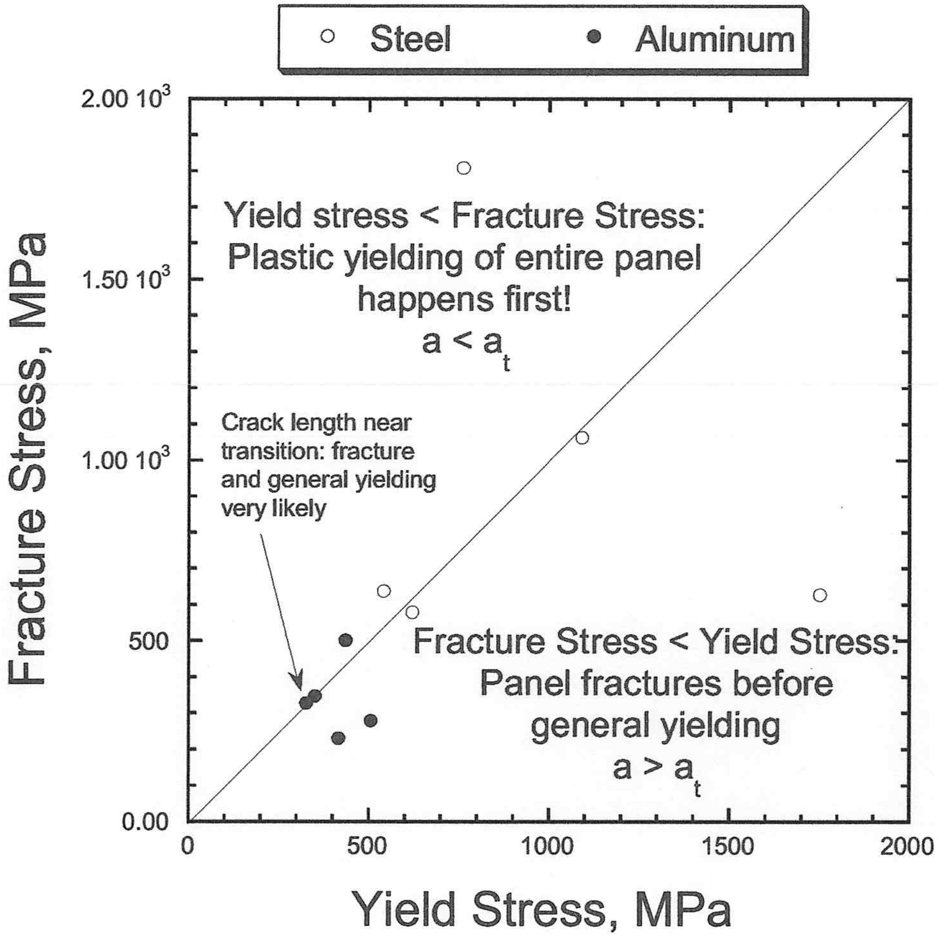
$$P_{fail} = A \cdot \sigma_{fail} = A \cdot \frac{K_{Ic}}{\sqrt{\pi a}} = 2.83x10^6\,N = 636{,}000\text{ lbs..}$$

This is a lot of force, not to mention figuring out a way to grip a panel 6'x6'. The solution? The invention of elastic-plastic fracture mechanics, which does not require the plastic zone to be small compared to other dimensions. In other words, you let large plastic zones happen (that may span the entire specimen), and use crack tip fields based on plasticity theory. The first to prove this works and explain how to measure toughness this way? James A. Begley and John D. Landes, in the early/mid 1970's.

Problem Four:

Here, I assume that a = 3.4 (i.e. the crack length is 2a = 6.8). I chose this simply because some of the materials would fail via fracture, and others via yielding. A plot is shown in the next page. The line indicates the condition that fracture stress (i.e. the stress needed to fracture the panel) is equal to the yield stress (i.e. the stress needed to yield the entire panel). When the fracture stress is larger than the yield stress, the panel will yield before breaking. When the yield stress is larger than the fracture stress, the panel will break before general yielding (although there will always be localized yielding near the crack). Points near the line indicate the flaw is near the transition flaw size, such that both fracture and general yielding are likely. Note that since the plastic zone size is approximately equal to the transition flaw sizes, points near the line involve plastic zones about equal to the given flaw size: hence, SSY does not apply and elastic-plastic fracture theories should be used. This makes perfect sense, since there will be lots of plasticity!

CE323 Fall 2003 Homework 5



November 20, 2003

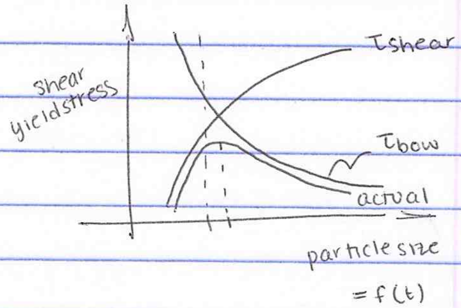
Homework help

1. make graph, trendline, Hall-Petch!
2. $\bar{V} = 4/3 \pi R^3$ $dV/dt = \text{const.}$

$$v(t) = V_0 + \frac{dV}{dt} \cdot t$$

find R(t)

plot shear stress vs. time



how does strength change when you anneal it?

max point is where they intersect / max combined curve

Concrete

November 25, 2003

SUPPOSEDLY, THIS IS THE LAST TOPIC AND THE HOMEWORK COMES DIRECTLY FROM IT.

Concrete materials

- cement ≡ inorganic material that reacts with water

  e.g. lime, Portland cement (pat. 1824)
- aggregate ≡ combination of gravel, sand, crushed stones, and slag

  (mineral crap off iron ore)
- concrete ≡ cement + aggregate + sand + H_2O
- mortar ≡ fine aggregate + cement + H_2O

  ↳ very fine

Cements – two categories

hydraulic cements – harden in H_2O

non-hydraulic cements – do not harden in H_2O

SiO_2 – silicon dioxide – "sand" ≡ S CaO – calcium oxide ≡ C

Al_2O_3 – aluminum oxide ≡ A Fe_2O_3 – iron oxide ≡ F

Typical composition of Portland cement

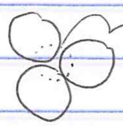
C~60-65%, S~20-25%, F, A~10%

$[3CaO] \cdot Al_2O_3 + 6H_2O \longrightarrow Ca_3Al_2(OH)_{12} + \text{heat}$ ⇝ rapid setting, low strength

$[2 \cdot C \cdot S + xH_2O \longrightarrow Ca_2SiO_4 + xH_2O + \text{heat}]$ high strength, slow setting

Sand : SiO_2, predominantly 0.05-1 mm diameter particles

- adsorbed H_2O involved
- acts to fill in gaps between large particles



full of sand instead of air; improves strength

- changes temperature response of the concrete

Aggregate

> clean, to promote bonding with cement

> strong, so as not to propagate cracks

> durable (tough)

* > size < 20% of the ~~thick~~ thickness of the structure

> shape – rectangular particles allow for interlocking geometries (strength increases)

however, stress concentrations are present, large surface areas (☹)

TABLE 17-5 ▣ *Types of Portland cements*

	Approximate Composition				
	$3C \cdot S$	$2C \cdot S$	$3C \cdot A$	$4C \cdot A \cdot F$	Characteristics
Type I	55	20	12	9	General purpose
Type II	45	30	7	12	Low rate of heat generation, moderate resistance to sulfates
Type III	65	10	12	8	Rapid setting
Type IV	25	50	5	13	Very low rate of heat gen
Type V	40	35	3	14	Good sulfate resistance

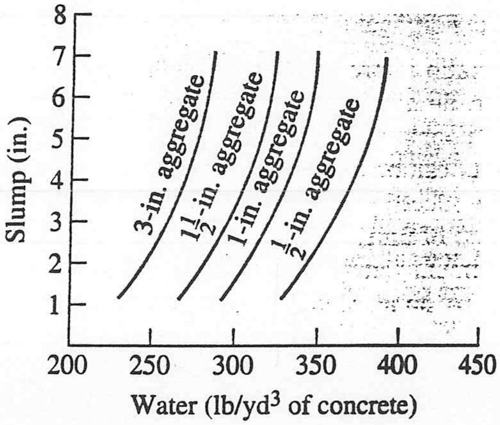


Figure 17-10
The amount of water per cubic yard of concrete required to give the desired workability (or slump) depends on the size of the coarse aggregate.

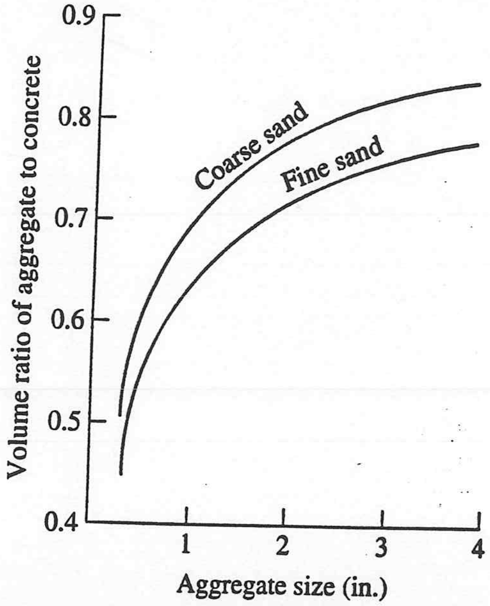


Figure 17-11
The volume ratio of aggregate to concrete depends on the sand and aggregate sizes. Note that the volume ratio uses the bulk density of the aggregate—about 60% of the true density.

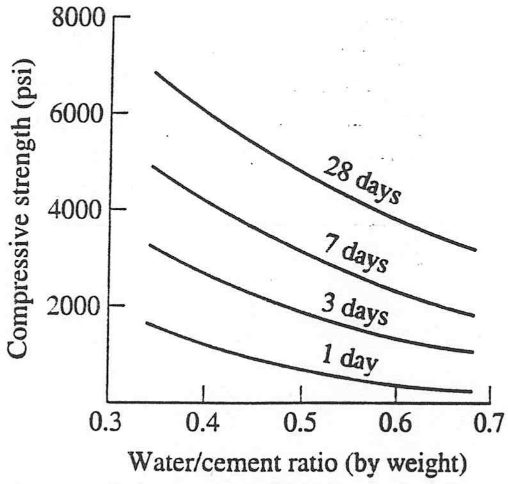


Figure 17-7
The compressive strength of concrete increases with time. After 28 days, the concrete approaches its maximum strength.

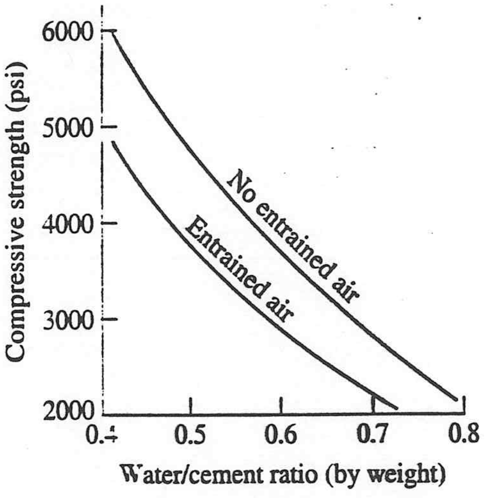


Figure 17-9
The effect of the water-cement ratio and entrained air on the 28-day compressive strength of concrete.

The Science and Engineering of Materials, D.R. Askeland, P.P. Phulé, Brookes/Cole Pub 2003 pg 711-30

Mystomach Hates Me

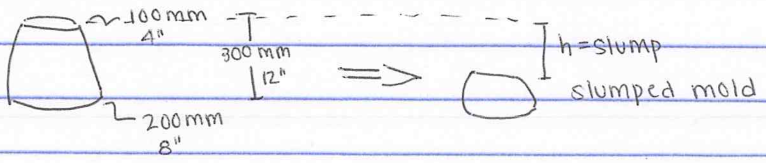
November 25, 2003

Water (as a material in cement)

- workability – H_2O improves the pourability, etc. (+)
- H_2O increases shrinkage (ha!), which is bad (-)
- decreases strength of concrete (-)

ENOUGH WATER TO MAKE IT MOVE, BUT NOT TOO MUCH TO MAKE IT WEAK.

Slump Test



$h \sim 1\text{-}6"$

Fill a cone, take mold off and ~~water~~ watch it slump

Air Entrainment

> mixing, pouring introduce air

> coarse aggregate (1.5" rock): 1% air by volume

fine aggregate (0.5" rock): 3% A.B.V.

deliberately add air to improve workability for very fine aggregates... up to 8% by volume

Aggregate content

– cost vs. strength ... slump vs. strength

Example: design a concrete mix for a retaining wall

GIVEN
$\sigma_c^{desired} = 4$ ksi thickness = 5 in ~~2% entrained air~~ aggregate: 1% moisture
coarsesand, 5% moisture $slump_{desired} = 3$ in height = 6 ft

1. choose aggregate size – 20% of thickness ~1%
2. water content – from figure, 320 lb H_2O / \$ yd^3
3. water-to-cement ratio

weight % H_2O / cement ~57%

cement = 561 lb/yd^3 concrete

Second-To-Last Class December 2, 2003

OH MY GOD. IF THIS WEREN'T THE LAST WEEK OF CLASS I THINK I'D HAVE TO GO KILL MYSELF.

Back to retaining wall example

strength is limited to workability and cost

- slump dictates water contents

  $320 \text{ lb } H_2O / yd^3$ - from 17.10

- strength dictates water-to-cement (w/cm) ratio

  0.57 - from Figure 17.9 $\left(\text{lb } \overset{H_2O}{\cancel{\text{cement}}} / \text{lb cement}\right)$

- amount of cement

$$\frac{\text{wt of cement}}{yd^3} = \frac{\text{lb } H_2O}{yd^3} \cdot \frac{\text{lb cement}}{\text{lb } H_2O} = 561 \text{ lb}/yd^3$$

- sand

  volume of aggregate : coarse sand, 1" aggregate = $0.7\ yd^3 \text{ agg}/yd^3 \text{ cement}$

$$\text{true volume} = \text{bulk volume} \cdot \underbrace{\rho_{true}/\rho_{bulk}}_{\sim 0.6}$$

  true volume = $0.42\ yd^3$ - Figure 17.11

- calculate volumes, as ft^3 —— $/yd^3$ concrete

  sand fills in whatever is left ($1\ yd^3 = 27\ ft^3$)

- typical units

  - water $7.48 \text{ gal}/ft^3$
  - cement $94 \text{ lb}/\text{sack}$
  - aggregate $170 \text{ lb}/ft^3$
  - sand $160 \text{ lb}/ft^3$

cement $190 \text{ lb}/ft^3$

LOOK AT HANDOUTS ON CONCRETE, ETC.

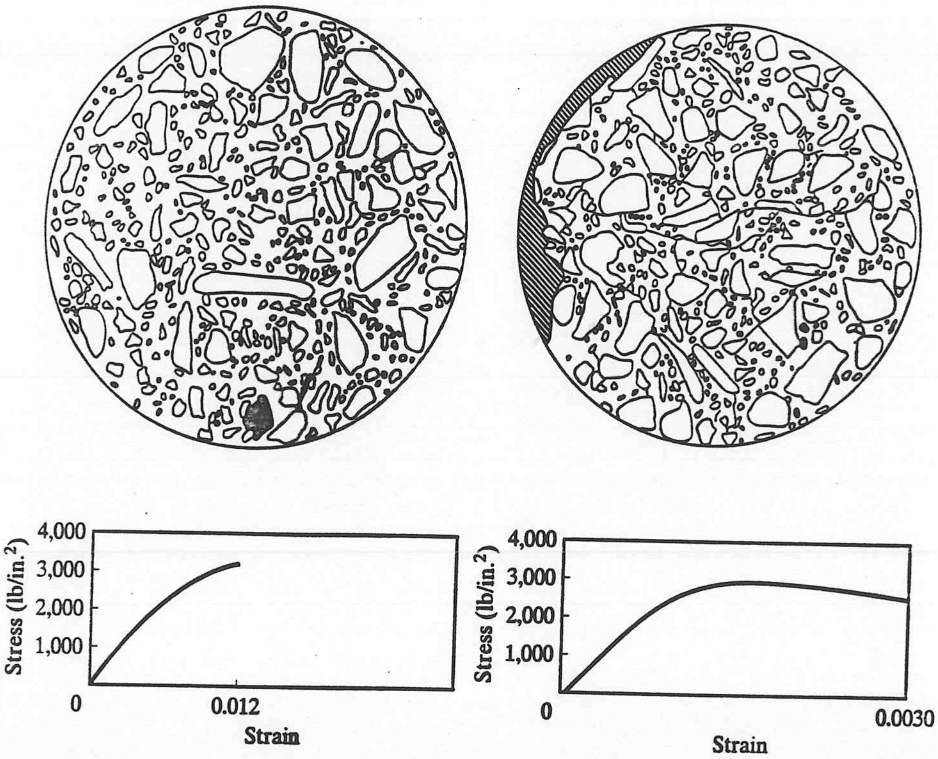


FIGURE 13.20

Cracking maps and stress–strain curves for concrete in uniaxial compression. [From S. P. Shah and F. O. Slate, in *The Structure of Concrete*, eds. A. E. Brooks and K. Newman, Cement and Concrete Association, London, pp. 82–92 (1968). Reproduced by permission of British Cement Association, formerly Cement and Concrete Association.]

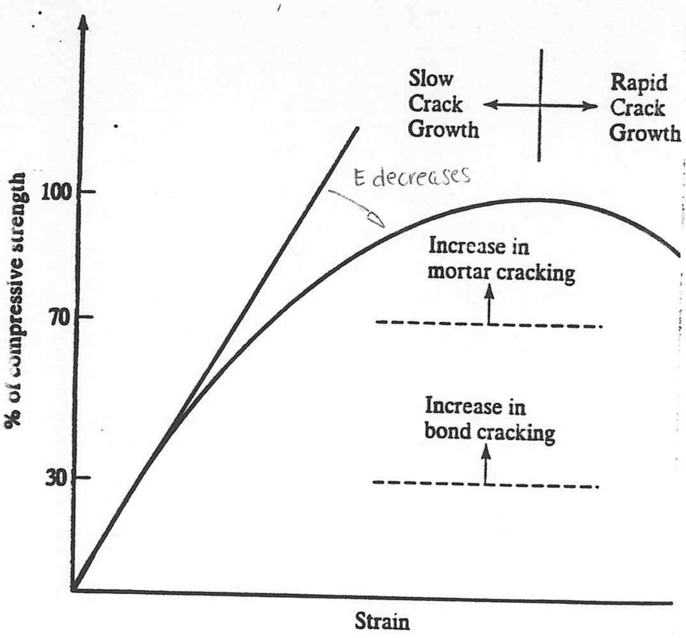


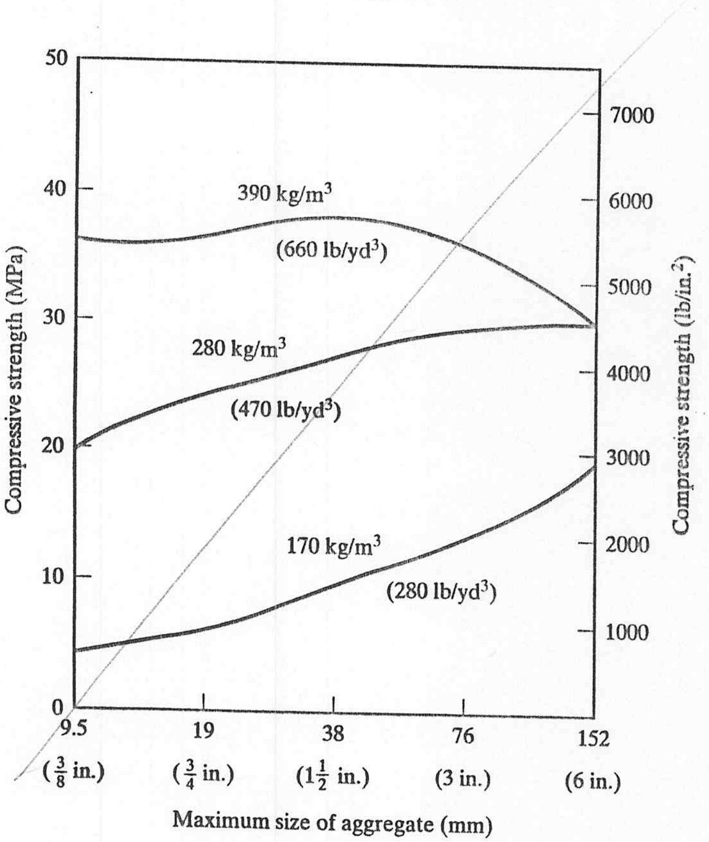
FIGURE 13.21

Diagrammatic stress–strain curve of concrete in compression. [From ACI Committee 224, *Journal of the American Concrete Institute*, Vol. 69, No. 12, pp. 717–753 (1972).]

- break bonds between aggregate, cement
- crush structure

(things are never so simple as gross generalities)

interfaces are bad – sites to initiate failure



URE 7.3

luence of aggregate size on 28-day mpressive strength of concretes with ferent cement contents. [From *nposium on Mass Concrete*, SP-6, nerican Concrete Institute, Detroit, , pp. 219–256 (1963).]

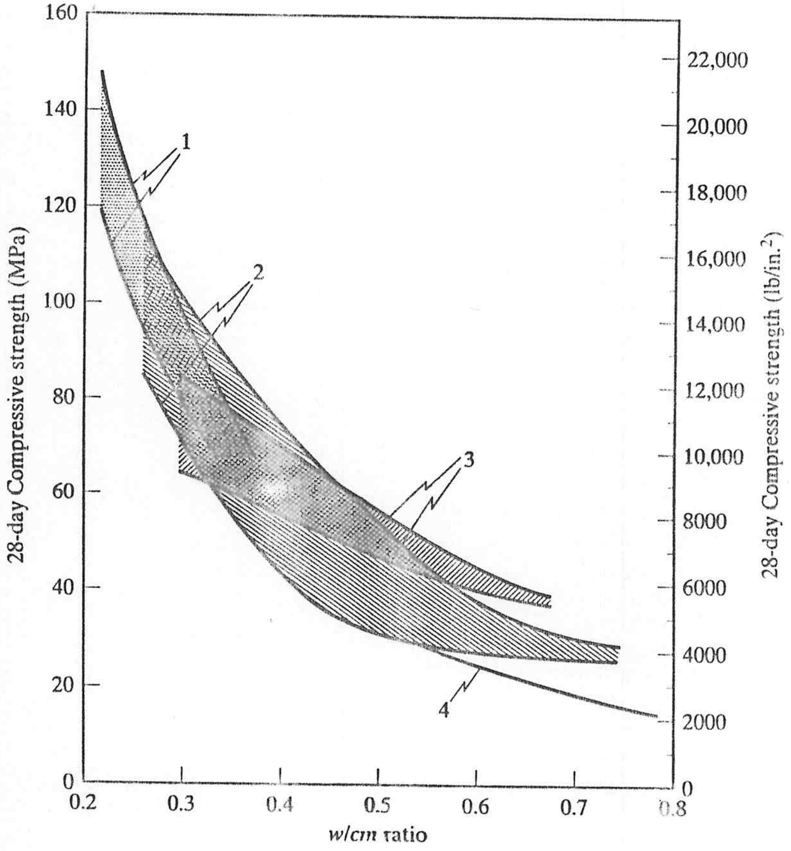
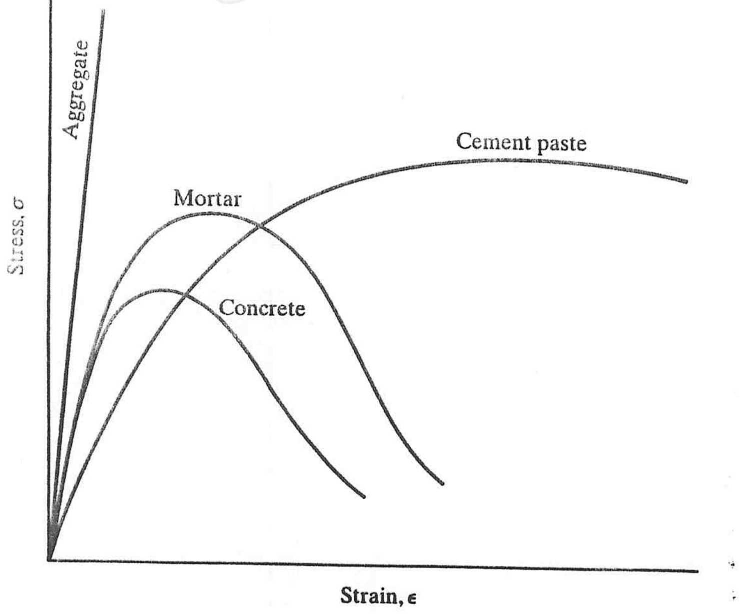
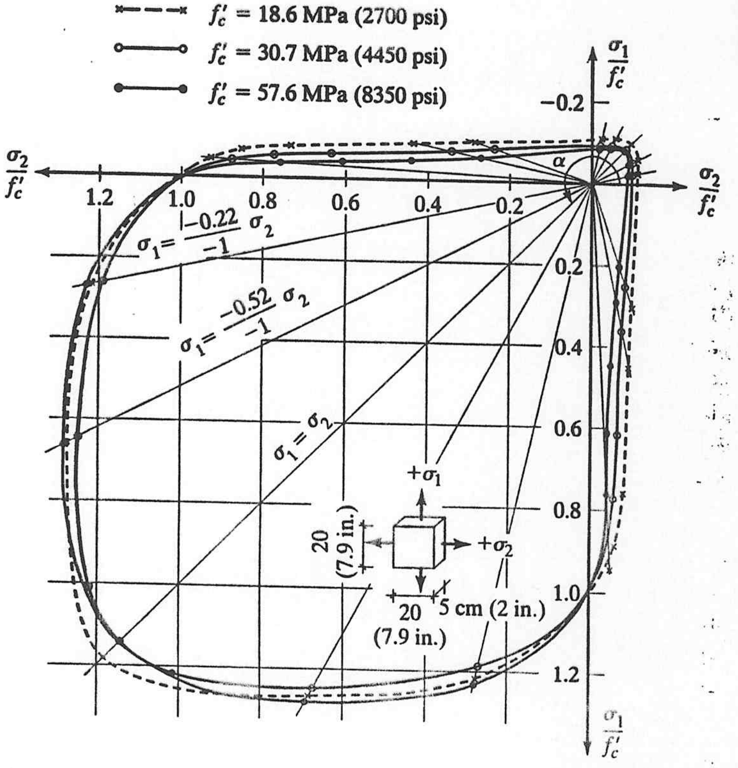


FIGURE 19.5

Compressive strength versus *w/cm* ratio for concretes from several sources. [Adapted from S. Mindess, in *High-Performance Concrete: Properties and Applications*, eds. S. P. Shah and S. H. Ahmad, McGraw–Hill, Inc., New York, p. 14 (1994).]

from Concrete, S. Mindess, J.F. Young & D. Darwin, Prentice Hall, 2003





GURE 13.38

axial strength of concrete. [Adapted om H. Kupfer, H. K. Hilsdorf, and . Rusch, *Journal of the American oncrete Institute*, Vol. 66, No. 8,). 656–666 (1969).]

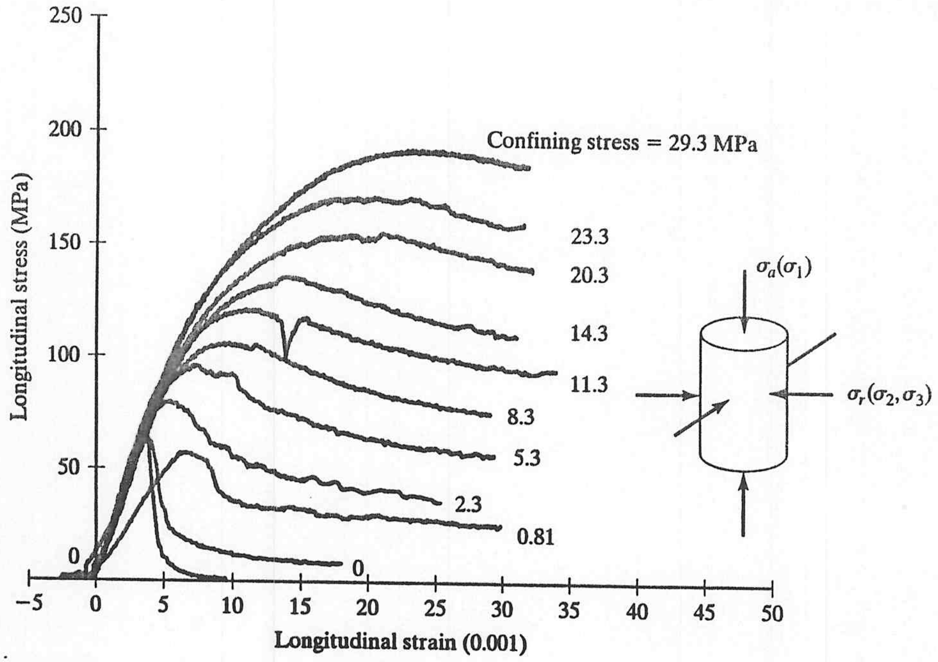


FIGURE 13.39

Longitudinal stress versus longitudinal strain for concrete under triaxial compression. [Adapted from J. Xie, A. E. Elwi, and J. G. MacGregor, *ACI Materials Journal*, Vol. 92, No. 2, pp. 135–145 (1995).]

from Concrete, by S. Mindess,
J. F. Young, &
D. Darwin

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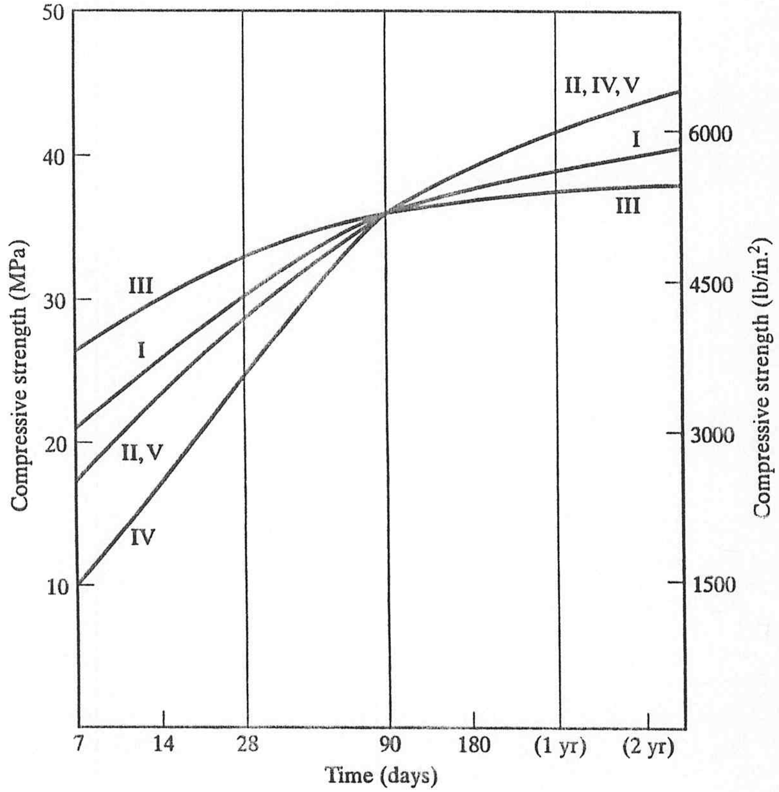

FIGURE 3.7

Strength of 6 × 2 in. (150 × 300 mm) concrete cylinders made with the same aggregate, but different cements. [Adapted from *Concrete Manual*, 8th ed., Bureau of Reclamation, Denver, CO (1975).]

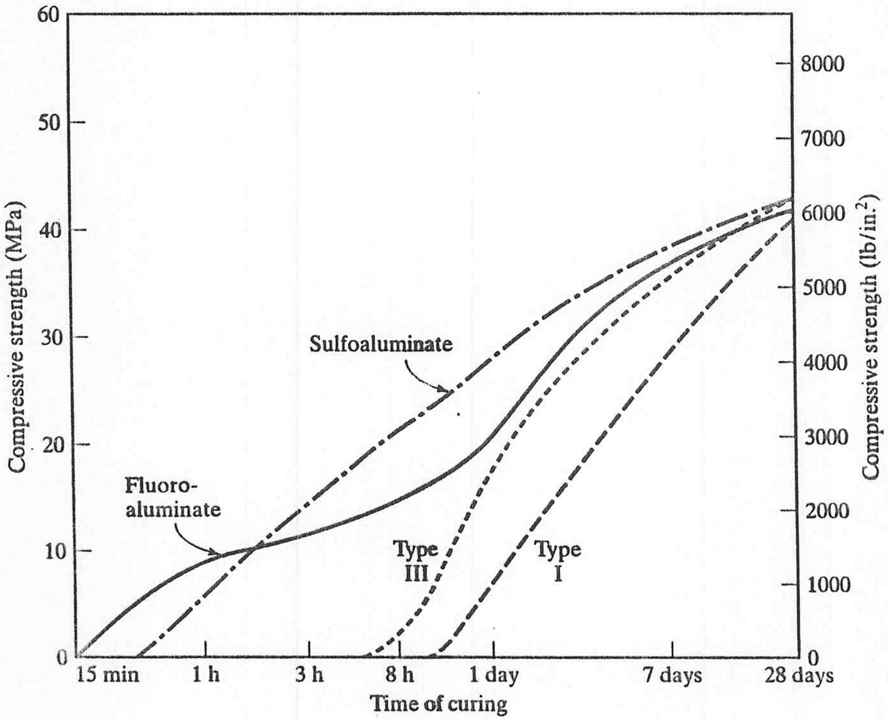

FIGURE 3.13

Strength development of concretes made with different rapid hardening cements. [Adapted from W. Perenchio in *New Materials in Concrete Construction*, University of Illinois at Chicago, p. 12-IV (1972).]

from Concrete, by S. Mindess, J.F. Young and D. Darwin
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TABLE 18.5 Causes of Cracking in Concrete Due to Interaction with Surroundings

Component	*Type*	*Cause of Distress*	*Environmental Factor(s)*	*Variables to Control*
Cement	Unsoundness	Volume expansion	Moisture	Free lime and magnesia
	Temperature cracking	Thermal stress	Temperature	Heat of hydration, rate of cooling
Aggregate	Alkali–silica reaction	Volume expansion	Supply of moisture	Alkali in cement, composition of aggregate
	D-cracking	Hydraulic pressure	Freezing and thawing	Absorption of aggregate, maximum size of aggregate
Cement paste	Plastic shrinkage	Moisture loss	Wind, temperature, relative humidity	Temperature of concrete, protection of surfaces
	Drying shrinkage	Moisture loss	Relative humidity	Mix design, rate of drying
	Sulfate attack	Volume expansion	Sulfate ions	Mix design, cement type, admixtures
	Thermal expansion	Volume expansion	Temperature change	Temperature rise, rate of change
Concrete	Settlement	Consolidation of plastic concrete around reinforcement		Concrete slump, cover, bar diameter
Reinforcement	Electro-chemical corrosion	Volume expansion	Oxygen, moisture	Cover, permeability of concrete

TABLE 18.6 Types of Cracking in Concrete Structures

Nature of Crack	*Cause of Cracking*	*Remarks*
Large, irregular, frequently with height differential	Inadequate support, overloading	Slabs on ground, structural concrete
Large, regularly spaced	Shrinkage cracking, thermal cracking	Slabs on ground, structural concrete, mass concrete
Coarse, irregular "map cracking"	Alkali–silica reaction	Extrusion of gel
Fine, irregular "map cracking" (crazing)	Excessive bleeding, plastic shrinkage	Finishing too early, excessive troweling
Fine cracks roughly parallel to each other on surface of slab	Plastic shrinkage	Perpendicular to direction of wind
Cracks parallel to sides of slabs adjacent to joints (D-cracking)	Excessive moisture contents, porous aggregates	Deterioration of concrete slab due to destruction of aggregates by frost
Cracks above and parallel to reinforcing bars	Settlement cracking	Structural slabs due to consolidation of plastic concrete around reinforcing bars near upper surface
Cracking along reinforcing bar placements, frequently with rust staining	Corrosion of reinforcement	Aggravated by the presence of chlorides

From Concrete, by S. Mindess, J.F. Young & D. Darwin
Prentice Hall 2003

Last Thursday!　　　　December 4, 2003

YIPPEE. SUPPOSEDLY WE'RE GOING TO TALK ABOUT THE HOMEWORK ... NEAT – LAST CLASS.

Concrete

vs. Steel – nonhomogenous material (boo)

good in compression, bad in tension

$\sigma_T = 8-15\% \ \sigma_c$

weaknesses

bleeding, honeycombing, air bubbles

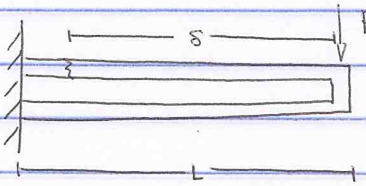
↳ water evaporating out of structure

↳ can be good ... can cause weakness

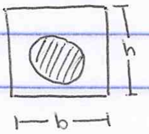
→ cracks growing through voids between aggregate

creep – time dependent deformation caused by loading

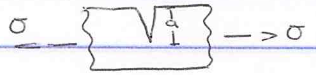
Composite



concrete beam with steel core



crack: $K_{edge} = 1.12\,\sigma\sqrt{\pi a}$

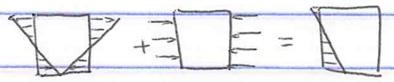


$$\sigma^{bend}_{edge} = \frac{Mc}{I} = \frac{6PS}{bh^2}$$

$$\sigma^{tot}_{edge} = \sigma^{bend} + \sigma^{comp.} = \frac{6PS}{bh^2} - \sigma_c$$

(–) because of sign convention

↳ comes from steel pulling concrete in



bend + comp = total

so $K_{Ic} = 1.12\sqrt{\pi a}\left(\frac{6PS}{bh^2} - \sigma_c\right)$　　$\sigma^{max}_{comp} = \sigma^c_f = \frac{6PS}{bh^2} + \sigma_c$

max load when failure by compression, fracture are equally likely

$\bar{P} = \frac{6PS}{bh^2}$　　$\boxed{\bar{P} = \frac{\bar{K}_{Ic} + \sigma^c_f}{2}}$　　$\bar{K}_{Ic} = \frac{K_{Ic}}{1.12\sqrt{\pi a}}$

load carrying capacity is a function of flaw size, K_{Ic}, σ^c_f

CE323 Properties and Behavior of Materials **Review for Final**
Fall 2003 **Friday, Dec. 12th**

Vocabulary terms that involve relatively straightforward definitions are underlined.

- **Material selection: performance indices for:**
    - bending stiffness for weight
    - buckling
    - strength for weight
    - energy storage
    - cost

- **Strength of materials:**
    - Hooke's law in three dimensions (with problems)
    - Elastic bending
    - Elastic-plastic bending

- **Material properties:**
    - Stiffness: elastic modulus
    - Strength: yield strength, ultimate tensile strength, fracture strength.
    - Toughness: work of fracture, critical energy release rate

---

QUIZ #1

- **Theoretical Strength of Materials:**
    - Stretching of inter-atomic bonds
    - Conversion of strain energy to surface energy
    - Atomic slip in crystals: no dislocations

- **Introduction to Defects and Failure Mechanisms:**
    - Circular holes
    - Elliptical holes: component strength vs. intrinsic material strength (theoretical)
    - Stress concentrations vs. stress singularities
    - Dislocations: slip planes, component yield vs. critical shear stress
    - Crushing: compressive loading of brittle materials
    - Cracking: conversion of strain energy to surface energy, work of fracture, fracture toughness, critical energy release rate

---

QUIZ #2

- **Brittle Failure: Maximum Normal Stress Criterion:**

  - Principal stresses
  - Mohr's Circle/Stress Transformations
  - Failure surfaces: anistropic yield/fracture criterion.

- **Ductile Failure**
  - Maximum shear stress criterion: Tresca yield criterion
  - Absolute maximum shear stress and out-of-plane yielding for 2-D stress states
  - Maximum shear yield (failure) surface (isotropic yield stress)
  - Von Mises yield criterion: formulae
  - Von Mises yield (failure) surface (isotropic yield stress)

- **Introduction to Defects and Failure Mechanisms:**
  - Circular holes
  - Elliptical holes: component strength vs. intrinsic material strength (theoretical)
  - Stress concentrations vs. stress singularities
  - Dislocations: slip planes, component yield vs. critical shear stress
  - Crushing: compressive loading of brittle materials
  - Cracking: conversion of strain energy to surface energy, work of fracture, fracture toughness, critical energy release rate

MIDTERM

- **Fracture Toughness and Linear Elastic Fracture Mechanics (LEFM)**
  - Stress fields near elliptical holes vs. cracks.
  - Stress fields near all cracks: the stress intensity factor
  - Fracture modes: I, II and III.
  - Critical stress intensity factors, or toughness: K_{Ic}
  - Critical energy release rate vs. critical stress intensity factors: Irwin's Relation
  - Plastic zone size estimates
  - Transition flaw sizes
  - Conditions for LEFM

- **Toughening Mechanisms:**
  - Damage at crack tips: metals, ceramics and polymers
  - Brittle fracture mechanism: role of hard particles
  - Ductile fracture: interplay of plastic deformation (and yield stress) on fracture toughness
  - Ductile fracture mechanism: micro-void coalescence

QUIZ #3

- **Yield stress/Strengthening Mechanisms:**
    - Review of dislocations as basis for metal plasticity
    - Introduction to dislocation interaction and strain hardening
    - Yielding of single crystals: Stage I, II and III hardening
    - Yielding of polycrystals: the Hall-Petch effect
    - Solid solution strengthening
    - Particulate/dispersoid strengthening: particle shearing
    - Particulate/dispersoid strengthening: dislocation bowing

QUIZ #4

- **Concrete**
    - Definitions of constituent materials
    - Types of cement: cement hydration reactions
    - Global perspective: strength vs. workability (slump)
    - Role of aggregate size on workability
    - Role of aggregate size on strength: interface area per unit volume
    - Desired water content for workability (slump)
    - Role of water-to-cement ratio on strength
    - Desire aggregate volume per cubic yard concrete
    - Role of entrained air on strength
    - Mix computations

    - Compressive stress-strain data: damage evolution
    - Compressive stress-strain: role of constraint
    - Reinforced concrete

FINAL

A few questions we've answered (in one way or another): non-inclusive!

- What is the property ratio of density and strength that will optimize the performance of a component loaded in _____? (E.g. for torsion, maximize $\sigma_f^{2/3}/\rho$.)
- What is the difference in response of a block of material loaded in compression if it does and does not touch restraining walls?
- What are the three keys of solid mechanics? What are three specific examples (formulae) that relate the key variables?
- How would you solve for an equation that can be used to extract elastic modulus from a beam loaded in bending?
- What is the load that will cause first yielding in a beam in bending? What is the load that will cause the entire beam to yield (i.e. the plastic collapse load)?
- How much of a beam yields for a given applied moment (i.e. what is the size of the elastic core)?
- Why can glass fibers exhibit incredible strength if they are made sufficiently small? What is an example (or proof) of this strength?
- What are the relative properties (stiff/flexible, strong/weak, brittle/tough) of: porcelain, a cracker, steel, nylon, Jell-O, pure copper?
- What is the failure mechanism of: ceramics, metals, concrete? How does one obtain an estimate for the strength of a defect-free material?
- How do real yield strengths compare with theoretical yield strengths predicted for a given slip plane orientation? Why?
- How does necking occur? Explain in terms of dislocation motion.
- What is the stress near a circular hole or elliptical crack? How does the stress vary with geometry of the defect?
- What is the difference between the strength of a test *specimen* and that of the material? What is the relationship between the failure stress of a component, the intrinsic material strength and a defect?
- What is the difference between a stress concentration and a stress singularity? Can you cite and explain an example that spans both concepts?
- What was Griffith's original fracture concept? What are the necessary modifications to his theory to make it work for brittle materials? For ductile materials?
- What are the relative toughness(es) of glass, ceramic, wood and metals?
- Do brittle materials fail in compression?
- For what type of materials is the maximum normal stress criterion used to predict failure?
- Given a two-dimensional stress state, compute the stresses acting in a coordinate system determined by a specified rotation from the original.
- For what type of materials is the maximum shear stress (or Tresca) criterion used to predict failure?
- What is meant by a yield (or failure) surface? Illustrate failure surfaces for the three stress-based approaches covered in class.

- What is meant by absolute maximum shear stress, and what are its implications for yielding in plane-stress problems?
- What combination of material properties should be maximized to make the strongest, lightest cantilever beam possible? How does this differ from the stiffest, lightest beam possible? What is the combination of material properties that should be maximized to make the strongest beam possible if the material has low fracture toughness?
- How does the stress vary near an elliptical hole, and how can this solution be used to illustrate the fundamental stress distribution near a crack?
- What are "damage mechanisms" near crack tips in metal, polymers and ceramic?
- What is meant by stress intensity factor, and how does it relate to the stresses at a crack tip?
- What is mean by the critical stress intensity factor, and how does it relate to toughness? How is it used to predict fracture according to linear elastic fracture mechanics?
- What are the ***two*** conditions that must be met to accurately apply LEFM? (
- What is meant by the transition flaw size, and how is it derived (approximately)?
- Why is it difficult to measure the fracture toughness of very ductile materials using LEFM? How is the plastic zone size estimated?
- What are the three fundamental modes of fracture? Which one is of greatest concern (usually) and why?
- What is the relationship between the strain energy release rate and stress intensity factors (for LEFM)?
- What is the role of (brittle) particle size on toughness for brittle modes of failure?
- What is the role of plastic deformation on toughness, and what does this imply regarding the general relationship between yield stress and toughness?
- What is the role of (brittle) particle size on toughness for ductile modes of failure?
- What is an appropriate performance index for designing a strong, lightweight beam comprised of a brittle material? Can you use this to explain why ceramics make lousy macroscale beams? (What about microscale beams?)
- How is the strain energy release rate determined for simple beam specimens – e.g. a beam with a crack down the axis and loaded with applied moments?
- Name five strengthening mechanisms and describe the global concept that underlies their function (in increasing yield stress).
- Describe the yielding of single crystals and explain the various hardening stages in terms of active slip systems and dislocation interactions.
- Which stages of hardening do polycrystals experience and why?
- What is the grain size effect in polycrystals?
- How can a fracture mechanics analogy be used to derive the relationship between yield stress and grain size?
- What is the Hall-Petch effect, and what are implications for material design?
- Why do solution solutions have higher yield strength (and/or strain hardening) than pure materials?
- What is the difference between a substitutional and interstitial solid solution? Which leads to greater increases in yield stress?

- What is the effect of particle size in precipitate/dispersoid strengthening?
- Why could one claim that big particles have relatively little effect on yield stress?
- Why could one claim that very small particles have relatively little effect on yield stress and toughness in ductile materials?
- What is meant by concrete, cement, aggregate and mortar?
- What is the difference between hydraulic and non-hydraulic cement?
- What is the underlying chemical reaction that leads to hardening in cement?
- Why might smaller aggregates lead to larger strength in concrete – support your answer with a quick back-of-the-envelope calculation?
- What is the global "trade-off" that should be considered when designing a cement mix? How does slump play a role?
- What is the relationship between water-to-cement ratio and the strength of concrete?
- What is the relationship between aggregate size and compressive strength in concrete? Does aggregate size have a greater or smaller effect than cement content?
- What increase in strength of concrete can be anticipated over a week, a month and year?
- What does a typical stress-strain curve look like for aggregate, mortar, concrete and pure cement?
- What is the effect of confinement (i.e. compressive stress perpendicular to the principal loading axis) on the strength of concrete? What is the underlying cause of this effect?
- What are the key microstructural features in wood? How do they each contribute to the tensile and compressive strength of wood?
- What is the effect of water content on the strength of wood?
- Can you cite three examples of "materials science tradeoffs", wherein increasing one property may decrease another?
- Can you briefly describe the "logic of the discipline" in the context of predicting material behavior?

CE323 Final Exam Formula Sheet

Fall 2003

$$\varepsilon_x = \frac{\sigma_x}{E} - v\left(\frac{\sigma_y}{E} + \frac{\sigma_z}{E}\right)$$

$$EI\frac{d^2v}{dx^2} = M(x)$$

$$\sigma(x,y) = \frac{M(x)y}{I}$$

$$\delta = \frac{Pl^3}{3EI}$$

$$\sigma_o = 2\sqrt{\frac{\gamma E}{a}}\ ,\ \sigma(\delta) = \sigma_o \sin\left(\frac{\pi\delta}{\delta_o}\right)$$

$$\sigma_{\max} = \sigma_\infty\left(1 + 2\frac{a}{b}\right) = \sigma_\infty\left(1 + 2\sqrt{\frac{a}{\rho_{tip}}}\right)$$

$$\sigma_{yy}(x, y=0) = \frac{\sigma_\infty a}{\sqrt{x^2 - a^2}}$$

$$\sigma_{11}' = \sigma_{11}\cos^2\theta + 2\sigma_{12}\sin\theta\cos\theta + \sigma_{22}\sin^2\theta$$

$$\sigma_{22}' = \sigma_{11}\sin^2\theta - 2\sigma_{12}\sin\theta\cos\theta + \sigma_{22}\cos^2\theta$$

$$\sigma_{12}' = \sigma_{12}\cos 2\theta + \frac{(\sigma_{22} - \sigma_{11})}{2}\sin 2\theta$$

$$\sigma_{VM} = \frac{1}{\sqrt{2}}\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_1 - \sigma_3)^2 + (\sigma_3 - \sigma_2)^2}$$

$$\sigma_{ij} = \frac{K}{\sqrt{2\pi r}} f_{ij}(\theta) \text{ where } f_{ij}(\theta \to 0) = 1$$

$$K = \sigma_\infty\sqrt{\pi a}$$

$$K = P/\sqrt{2bA}$$

$$G = \frac{6P^2a^3}{Eb^2h^3}$$

$$G = \frac{3E\Delta^2h^3}{2a^4}$$

$$G = \frac{K_I^2}{E} + \frac{K_{II}^2}{E_*} + \frac{K_{III}^2}{E_*}$$

$$r_p \approx \frac{1}{2\pi}\left(\frac{K}{\sigma_y}\right)^2$$

$$a_t = \frac{1}{\pi}\left(\frac{K_{Ic}}{\sigma_y}\right)^2$$

$$\tau_I = \tau_p + \alpha_I G b\sqrt{\rho}$$

$$\Delta\sigma = G\beta(\varepsilon)\sqrt{C}$$

$$\sigma_y = \sigma_o + k_y d^{-1/2}$$

$$\tau_{BOW} \approx \frac{1}{2} G b \frac{\sqrt{f}}{R}$$

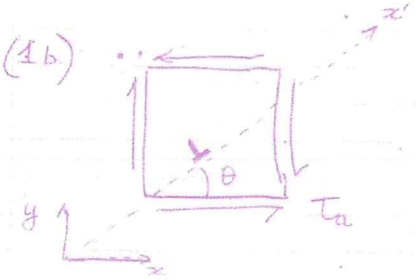
$$\tau_{SHEAR} \approx 3G\left(\frac{\gamma_{AFB}}{Gb}\right)^{3/2}\sqrt{\frac{fR}{b}}$$

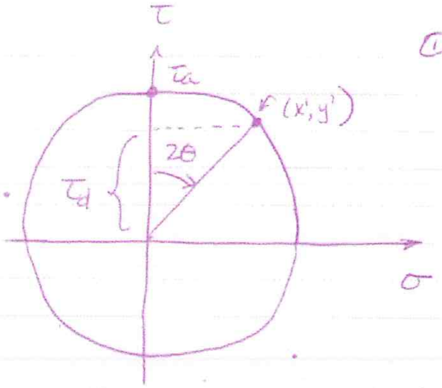
Exam will include select handouts that may or may not be needed.

CE223.1 FALL 2003
QUIZ #2 SOLUTIONS

①

(1b)



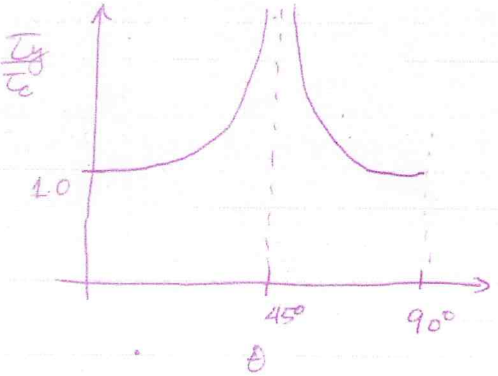


$\tau_d = \tau_a \cos 2\theta$ shear stress acting on dislocation

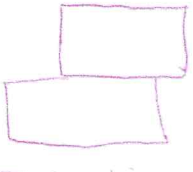
when $\tau_d = \tau_c$ critical shear stress to move dislocation, yield happens.

$\tau_y = \dfrac{\tau_d}{\cos 2\theta}$

At θ = 45°, there is no shear on dislocation, only normal stress!



(1c)



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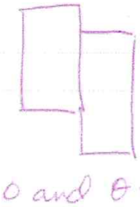

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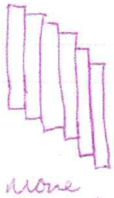


Dislocations at θ = 0 and θ = 90 will move first. other dislocations move at higher loads.

CE231 FALL 2003

②

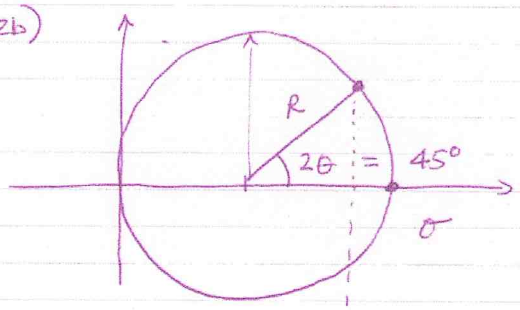
(2a) $\frac{\pi \sigma^2 a}{E} = G_c$; $\sigma = \frac{P}{A} = \frac{P}{\pi r^2}$

$$a = \frac{E G_c}{\pi \sigma^2} = \frac{220}{\pi (0.003)^2} \frac{N}{m^2}$$

$$a = \frac{(70\times10^9)(2)}{\pi (7.8\times10^6)^2} \Leftarrow \sigma = 7.8\times10^6$$

$a = 732\ \mu m$ ($2a \sim 1.5\ mm$)
total crack length.

(2b)



$R = \frac{\sigma}{2}$

$$\sigma_{crack} = \frac{\sigma}{2} + \frac{\sigma}{2} \cos 2\theta$$

$$\sigma_{crack} = \sigma (0.5 + 0.354) = 0.854\, \sigma_{applied}.$$

$$\sigma_{crack} = \sqrt{\frac{E G_c}{\pi a}} = \sqrt{\frac{(70\times10^9)(2)}{\pi (880\times10^{-6})}}$$

CE323.1 Quiz #2

(2b) cont'd. ③

$\sigma_{crack} = 23.6 \text{ MPa}$

$\sigma_{applied} = \frac{23.6}{0.854} \text{ MPa} = \boxed{27.64 \text{ MPa} = \sigma_{app}^{fail}}$

$P = \pi(0.003)^2 \cdot 27.64 \text{ MN} = 781 \text{ N}.$

(1a) We can see from the results of 1b that the specimen fails when the stress on the defect reaches a critical value: the stress on the defect will be different from stress applied to specimen. Generally, for ⊥ motion,

$\tau_{defect} < \tau_{applied}$ (unless ⊥ is at $\theta = 0, 90°$)

So that:

$\tau_{material}^{fail} < \tau_{specimen}^{fail}$

This simply says that unless defect is unfortunately aligned, not all of the applied stress is transmitted to the slip plane.

This is in contrast to brittle failure near a hole or crack, where maximum stress is many times greater than the applied stress. In this scenario,

$\sigma_{max} > \sigma_{applied}$

due to stress concentration (say at time of elliptical hole). So, failure happens when

$\sigma_{max} = \sigma_{material}^{fail}$

So:

$\sigma_{mat'l}^{fail} > \sigma_{applied}^{fail}$

Thus, failure stress of component is allways less than failure of mat'l, since stress concentrations raise stresses locally.

(2c) The energy of fracture (or strain energy released) is converted to surface energy and absorbed in material via heat, plastic flow, frictional sliding. <u>But</u>,

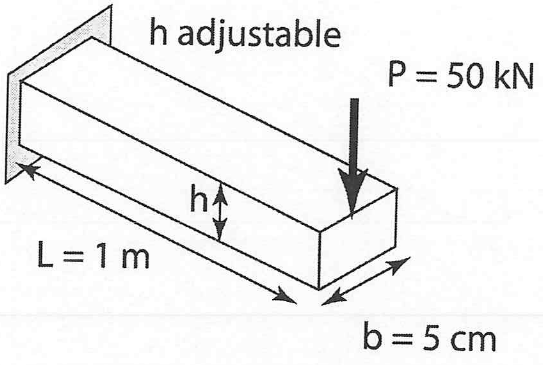
$G_c \sim \gamma_{surface}$, so theren't

really any dissipation mechanisms.

CE323 Properties and Behaviors of Materials
Fall 2003

MIDTERM
October 28th

Problem One:



a. Determine the combination of material properties that should be optimized if one wishes to build the **strongest**, **lightest** cantilever beam possible – i.e. derive the appropriate performance index for the scenario above.

b. Using the relevant material map, identify two very different classes of material that have similar and desirable performance indices.

c. Consider making two beams, one from each of the materials identified above. How do the final designs differ? Be as quantitative as possible. Briefly comment on the practicality of each material choice.

Problem Two:

Consider a brittle material and a ductile material ~~*with a single dislocation:*~~

a. Briefly explain the mechanism of "failure" for each material.

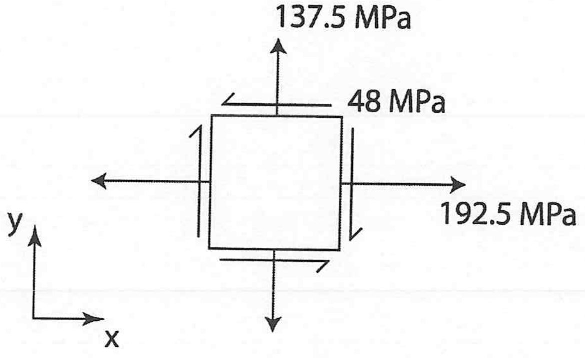
b. ~~Identify the relationship between the stress needed to fail a specimen and the stress needed to trigger the appropriate "failure" mechanism, for each material. (I.e., explain how these relationships may be different for a brittle material and a ductile material with a single dislocation.)~~

c. Briefly explain why these two materials have vastly different fracture toughness.

d. Briefly comment on the implications of the different fracture toughness(es) on the critical flaw sizes that lead to failure; if possible, use a numerical example to illustrate your comment(s).

Problem Three:

The yield stress of a ductile metal is $\sigma_y = 200\,\text{MPa}$. The stress state in a component is shown on the stress element below:

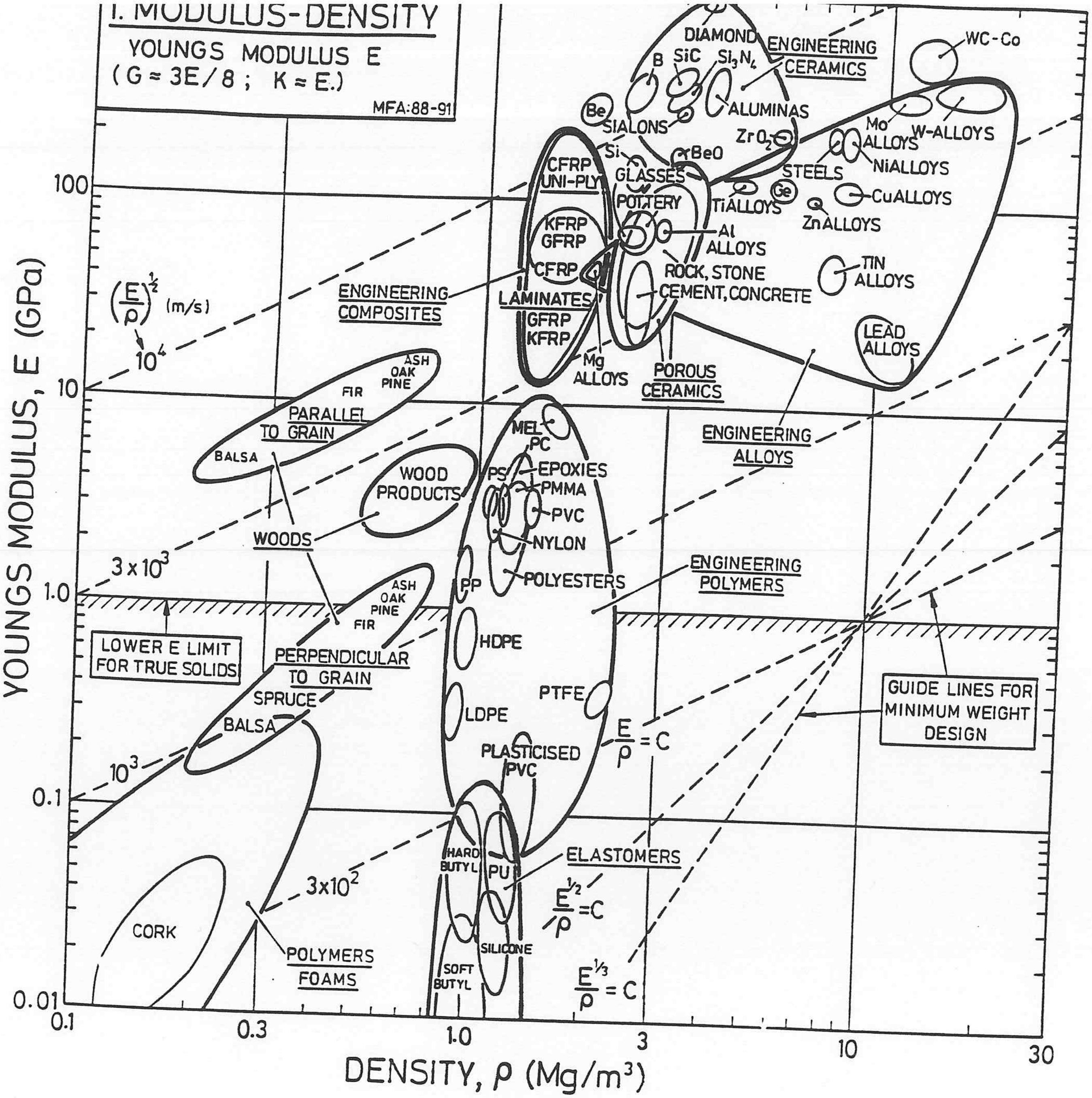


a. Sketch the stresses acting on a material element aligned with the principal axes.

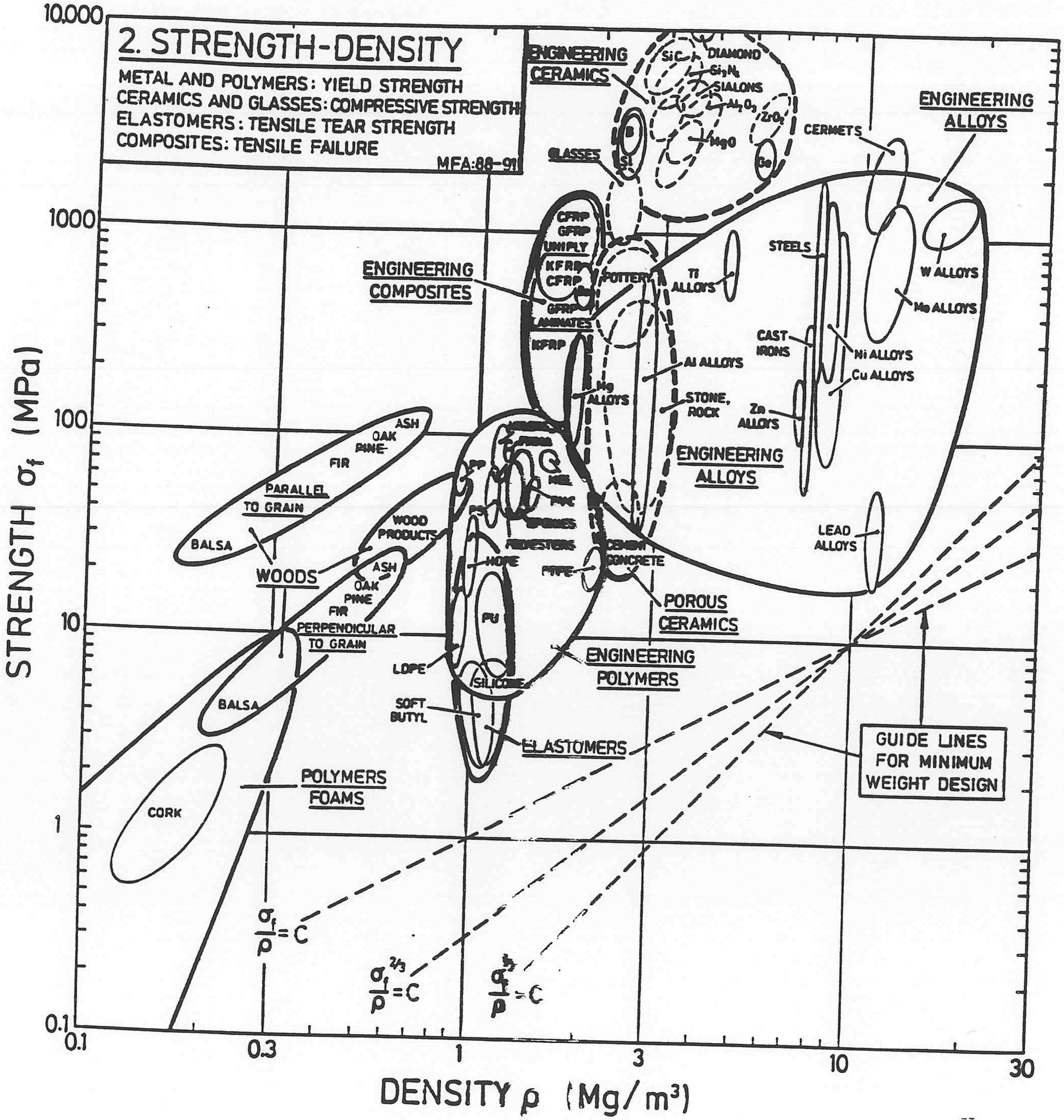
b. Determine if the material yields, according to the maximum shear stress criterion and comment on the direction of yielding.

c. Sketch the appropriate yield surface and the point that represents the given stress state.

d. Determine if the material yields according the Von Mises criterion; clearly justify your conclusion.



1. MODULUS-DENSITY
YOUNGS MODULUS E
(G ≈ 3E/8 ; K ≈ E.)
MFA:88-91
YOUNGS MODULUS, E (GPa)
DENSITY, ρ (Mg/m³)
0.01
0.1
1.0
10
100
0.1
0.3
1.0
3
10
30
$(E/\rho)^{1/2}$ (m/s)
10^4
3×10^3
10^3
3×10^2
DIAMOND
ENGINEERING CERAMICS
B
SiC
Si_3N_4
Be
SIALONS
ALUMINAS
ZrO_2
Si
BeO
GLASSES
POTTERY
CFRP UNI-PLY
KFRP GFRP
CFRP
LAMINATES
GFRP KFRP
ENGINEERING COMPOSITES
Mg ALLOYS
POROUS CERAMICS
ROCK, STONE
CEMENT, CONCRETE
Al ALLOYS
Ti ALLOYS
Ge
STEELS
Zn ALLOYS
WC-Co
Mo ALLOYS
W-ALLOYS
Ni ALLOYS
Cu ALLOYS
TIN ALLOYS
LEAD ALLOYS
ENGINEERING ALLOYS
ASH
OAK
PINE
FIR
PARALLEL TO GRAIN
BALSA
WOOD PRODUCTS
WOODS
ASH
OAK
PINE
FIR
PERPENDICULAR TO GRAIN
SPRUCE
BALSA
LOWER E LIMIT FOR TRUE SOLIDS
MEL
PC
PS
EPOXIES
PMMA
PVC
NYLON
PP
POLYESTERS
ENGINEERING POLYMERS
HDPE
PTFE
LDPE
PLASTICISED PVC
GUIDE LINES FOR MINIMUM WEIGHT DESIGN
$\frac{E}{\rho} = C$
ELASTOMERS
HARD BUTYL
PU
$\frac{E^{1/2}}{\rho} = C$
SILICONE
SOFT BUTYL
$\frac{E^{1/3}}{\rho} = C$
CORK
POLYMERS FOAMS


2. STRENGTH-DENSITY
METAL AND POLYMERS: YIELD STRENGTH
CERAMICS AND GLASSES: COMPRESSIVE STRENGTH
ELASTOMERS: TENSILE TEAR STRENGTH
COMPOSITES: TENSILE FAILURE
MFA:88-91
ENGINEERING CERAMICS
SiC
DIAMOND
Si_3N_4
SIALONS
Al_2O_3
ZrO_2
MgO
B
Si
Ge
GLASSES
CERMETS
ENGINEERING ALLOYS
W ALLOYS
Mo ALLOYS
STEELS
Ti ALLOYS
CFRP
GFRP
UNIPLY
KFRP
CFRP
GFRP
LAMINATES
KFRP
ENGINEERING COMPOSITES
POTTERY
CAST IRONS
Ni ALLOYS
Cu ALLOYS
Al ALLOYS
Mg ALLOYS
STONE, ROCK
Zn ALLOYS
ENGINEERING ALLOYS
ASH
OAK
PINE
FIR
PARALLEL TO GRAIN
BALSA
WOODS
WOOD PRODUCTS
PP
PS
MEL
PVC
EPOXIES
POLYESTERS
HDPE
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CONCRETE
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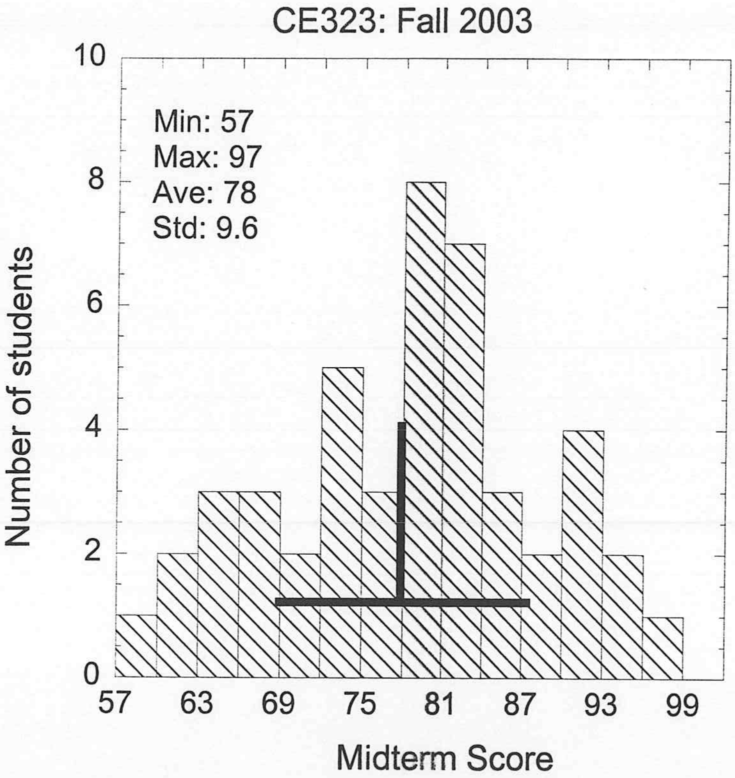
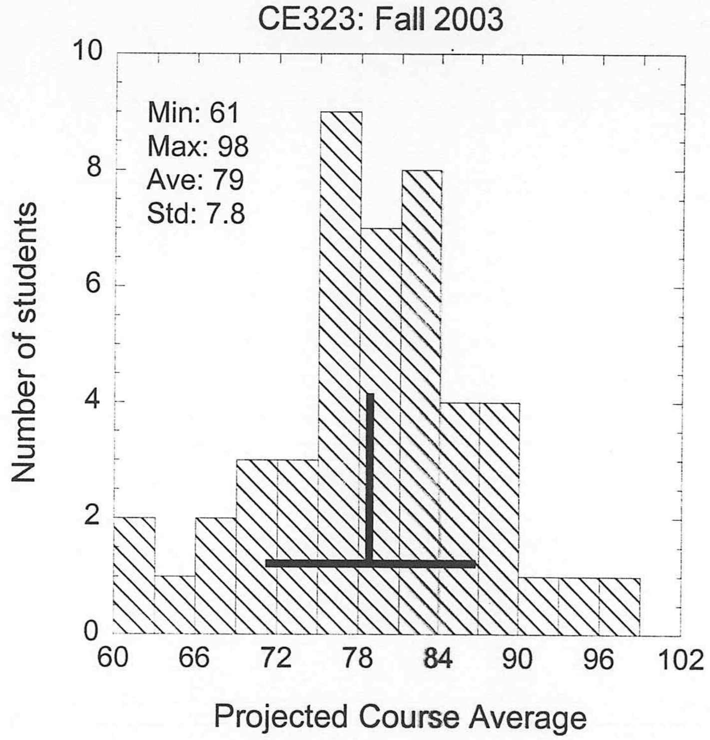
PERPENDICULAR TO GRAIN
PU
LDPE
SILICONE
ENGINEERING POLYMERS
BALSA
SOFT BUTYL
ELASTOMERS
GUIDE LINES FOR MINIMUM WEIGHT DESIGN
POLYMERS FOAMS
CORK
$\frac{\sigma_f}{\rho} = C$
$\frac{\sigma_f^{2/3}}{\rho} = C$
$\frac{\sigma_f^{1/2}}{\rho} = C$
STRENGTH σ_f (MPa)
10.000
1000
100
10
1
0.1
0.1
0.3
1
3
10
30
DENSITY ρ (Mg/m³)





Midterm: 1a 10pts 2a 10 pts 3a 10pts
1b 5pts 2c 5pts 3b 5pts
1c 5pts 2d 5pts 3c 5pts
3d 5pts

avg. 47.6

} 65 pts: best 61 taken as score:

$\frac{\text{Your score}}{61} \times 100 = \%$

CE323 Properties and Behaviors of Materials **Quiz 3**
Fall 2003 **November 20th**

Problem One:

Explain what is meant by the transition flaw size and how it factors into the analysis/design of specimens for measuring toughness. Clearly identify the material properties that control the transition flaw size.

Ans. The transition flaw size is that such that the fracture stress is equal to the yield stress. For a center cracked panel:

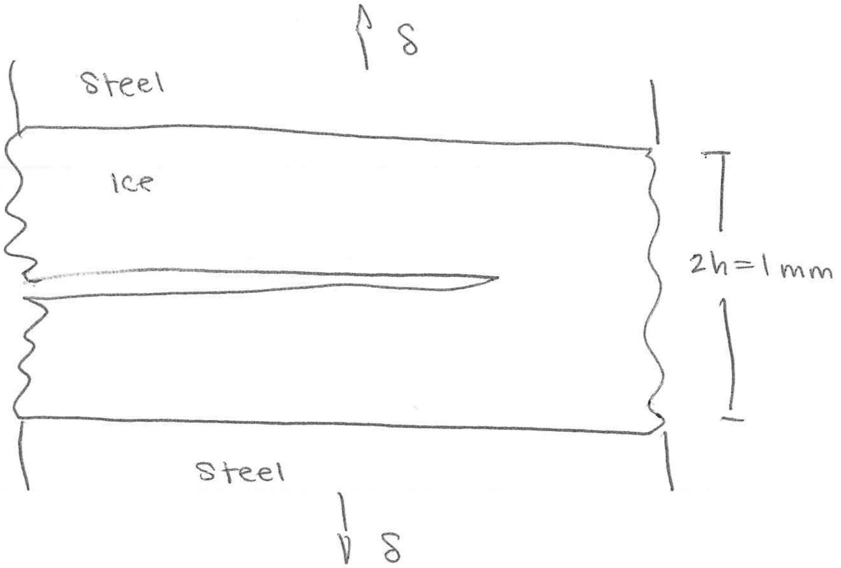
$K = \sigma\sqrt{\pi a}$, and at failure $K = K_{Ic}$, so that $\sigma_{frac} = \dfrac{K_{Ic}}{\sqrt{\pi a_t}} = \sigma_y$. Solving for the flaw size, one obtains: $a_t = \dfrac{1}{\pi}\left(\dfrac{K_{Ic}}{\sigma_y}\right)^2$. The transition flaw size thus represents the boundary between fracture based failure and yielding; flaw size less than the transition flaw size imply that the component yields before fracture, while flaw sizes larger than the transition flaw size indicate fracture occurs before general yielding.

To measure toughness, general yielding should be avoided if possible. This means that one desires a small transition flaw size, such that a relatively small crack will lead to fracture and enable the toughness measurement. Otherwise, large flaws will be needed to avoid general yielding, in turn implying large specimens which are difficult to test. This result is echoed by the estimated plastic zone size, which varies with toughness and yield strength in the same manner. Large transition flaw sizes mean large plastic zones, requiring large specimens such that LEFM can be applied.

Problem Two:

A thin layer of ice fills the gap between two moving steel pieces of a drawbridge and bonds them together (ice sticks devilishly to steel). A crack that is much longer than the gap size (a >> h) is running through the center of the strip of ice, as shown below.

The stress intensity factor for this scenario is: $K = \dfrac{E\delta}{\sqrt{(1-v^2)h}}$, where δ is the displacement of one side. The elastic modulus of ice is $E \sim 9$ GPa. The Poisson's ratio is $v = 0.25$. The fracture toughness is $K_{Ic} \sim 0.5$ MPa $m^{1/2}$. The yield stress is $\sigma_y = 85$ MPa. The total gap size is: $2h = 1$ mm.



a. Compute the relative displacement of the two pieces of bridge needed to advance the crack.

Ans. $\delta_{frac} = \frac{K_{Ic}}{E}\sqrt{(1-v^2)h} = \frac{5x10^5\,Pa\sqrt{m}}{9x10^9\,Pa}\sqrt{(1-0.25^2)5x10^{-4}\,m} = 1.2\mu m$. This is really small – small relative motions will crack the ice.

b. If the two pieces of the bridge move just enough to start the crack and stopped, does the crack stop or continue one it begins to propagate – explain your answer.

Ans. Once the crack is much longer than the thickness of the ice, the stress intensity factor becomes dependent only on ice thickness, as indicated above. Hence, once the crack advances, the energy release rate remains constant, and the stress intensity factor is still equal to the critical stress intensity factor, so the crack never stops.

c. Were you justified in using LEFM? Justify your answer with a quick calculation.

Ans. The requirement for LEFM is that the plastic zone size is much smaller than all other dimensions. The plastic zone size can be estimated as:

$$r_p \approx \frac{1}{2\pi}\left(\frac{K_{Ic}}{\sigma_y}\right)^2 = \frac{1}{2\pi}\left(\frac{5x10^5\,Pa\sqrt{m}}{8.5x10^7\,Pa}\right)^2 = 5.5\mu m << h$$

Since h is the smallest dimension, the plastic zone is small compared to all other physical length scales, so LEFM is correctly applied.

d. Are there any toughening mechanisms at work in this ice? Justify your answer.

Ans. This is determined by comparing the fracture toughness – in terms of energy – to the surface energy of the solid. Using Irwin's relation, the fracture toughness can be converted to the critical energy release rate:

$G_c = \frac{(1-v^2)K_{Ic}^2}{E} = \frac{(1-0.25^2)(5x10^5\,MPa\sqrt{m})^2}{9x10^9\,MPa} = 26\frac{J}{m^2}$. Note that $\gamma_{H_2O} \sim 1\frac{J}{m^2}$, $G_c^{ceramic} \sim 5-10\frac{J}{m^2}$, and $G_c^{wood} \sim 50-100\frac{J}{m^2}$. Hence, there are energy dissipation mechanisms (i.e. toughening mechanisms), and they are not negligible – ice is tougher than ceramic. Though smaller, they are comparable to toughening mechanisms in wood. Of course, they are not nearly as efficient as those in metal, where plastic dissipation increases toughness over surface energy by 4-6 orders of magnitude.

e. What is the maximum tensile stress developed in the intact portion of the ice, far from the crack tip?

Ans. The stress far from the crack is uniform, and a simple tension specimen:

$$\varepsilon = \frac{\delta}{h} = \frac{(1-v^2)\sigma}{E}\text{; this implies } \sigma_{frac} = \frac{K_{Ic}}{\sqrt{(1-v^2)h}}.$$

Rearranging the stress intensity factor, or via direct substitution, one obtains: $\sigma_{frac} \approx 23MPa$. This is not inconsequential: if there is a flaw in the steel, which is cold, it may trigger failure of the steel component first! This happens all the time in off-shore oil rigs: cracks in ice adhered to structures lead to relative movement that ruptures steel: the bonding of ice to steel in such scenarios is particularly troublesome.

What if the ice is much thinner, say 0.001 mm? Comment on the implications of your answer.

$$\sigma_{frac} = \frac{K_{Ic}}{\sqrt{(1-v^2)h}} = 516\text{ MPa} – \text{this a large stress.}$$

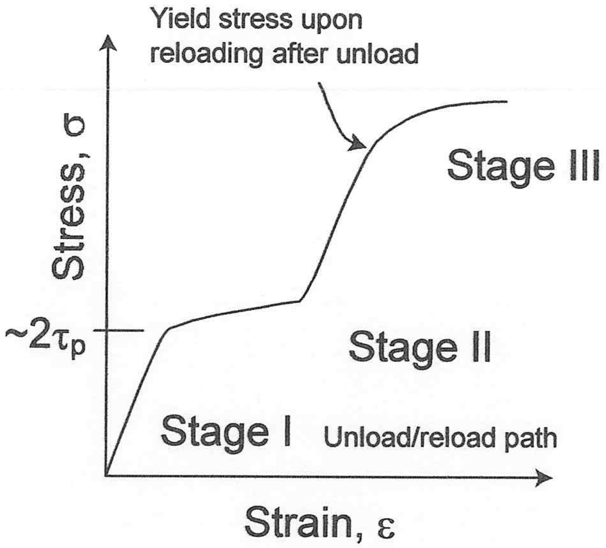
It implies that thin layers of ice bonding steel are incredibly troublesome. The thickness of the ice acts as the flaw size; small thickness implies the ice is very strong, and you can wind up breaking the steel first! BUT: in this scenario the estimated plastic zone is bigger than the ice thickness, implying elastic-plastic fracture is needed for the ice. Moral: thin ice is a difficult problem!

CE323 Properdties and Behaviors of Material s **Quiz 4**
Fall 2003 **December 2nd**

SOLUTIONS

Problem One:

a.) Describe the yielding behavior of metallic single crystals and explain the various hardening stages in terms of active slip systems and dislocation interactions.



Single crystals experience three stages of hardening:

Stage I: slip initiates on a single slip system with the largest resolved shear stress; the minimum possible uni-axial stress to cause yield is $\sigma_y = 2\tau_p$, where τ_p is the intrinsic lattice resistance, or Peierl's stress (previously discussed as the critical shear stress needed to move a dislocation). Typically, $\sigma_y = (2-3)\tau_p$. Additional dislocations are created, but slip is still confined to a single slip plane, so the rate of strain hardening, or slope of the stress-strain curve is relatively small, i.e. $\frac{d\sigma}{d\varepsilon} \sim 0.001E$. In terms of shear stress acting on a slip plane, the yield stress can be written as:

$$\tau_I = \tau_p + \alpha_I G b \sqrt{\rho}$$

where *G* is the material's shear modulus, *b* is the Burger's vector of a dislocation (a measure of lattice distortion caused by the dislocation) and ρ is the density of dislocations (measured as the number of dislocations passing through a unit area). A typical value for the constant $\alpha_I \sim 0.05$. Typical dislocation densities and increases in shear yield stress are:

annealed: $\rho \sim 10^7$ cm^{-2}, $\Delta\tau = \tau_I - \tau_p \sim 0.1MPa$
cold worked: $\rho \sim 10^{11}$ cm^{-2}, $\Delta\tau = \tau_I - \tau_p \sim 10MPa$

Since slip occurs on a single slip system, you cannot get high yield strengths because dislocations do not strongly interact (which reduces their mobility).

Stage II: in this stage, multiple slip occurs on different slip planes, and greatly increases the number of dislocations interactions that reduce mobility. In this stage, slip of dislocations on one plane is inhibited by forest dislocations threading through a perpendicular plane. The shear yield stress can again be written as:

$$\tau_{II} = \tau_p + \alpha_{II} Gb\sqrt{\rho}$$

where the hardening constant is $\alpha_{II} \sim 0.2 - 0.4 >> \alpha_I$. From the above, this leads to increases in yield strength on the order of 60 MPa, which is comparable to the intrinsic lattices resistance. Hence, the yield stress during this stage of hardening may be double the initial yield stress.

Stage III: in this stage, extensive cross slip and high dislocation densities result in dislocation annihilation, which limits the dislocation density that can be achieved. Since the dislocation density saturates (additional deformation doesn't significantly increase dislocation density since newly created dislocations are "cancelled" via annihilation), there is no significant increase in yield stress.

b.) *Describe the yielding behavior of polycrystals in the context of your response to part (a). What is the role of grain boundaries?*

Polycrystals do not experience Stage I hardening, since multiple slip occurs at the onset of yield due to random grain orientation. Stage II happens immediately, and eventually Stage III does occur. Grain boundaries act as obstacles to dislocation motion, increasing the yield stress. Smaller grains have more grain boundaries (surfaces) per unit volume, implying that there is a greater density of obstacles: hence, smaller grains mean larger yield strengths. This is described by the Hall-Petch relation, which predicts that yield stress will increase with grain size according to:

$$\sigma_y = \sigma_o + k_y / \sqrt{d}$$

For face-centered cubic (FCC) crystals, $k_y \sim 0.05 - 0.1$ MN m$^{-3/2}$, which for body-centered cubic (BCC) and hexagonal close-packed (HCP) crystals, $k_y \sim 0.2 - 0.4$ MN m$^{-3/2}$.

Problem Two:

a.) Why could one claim that very big particles/inclusions have relatively little effect on yield stress? What are the conditions that are required for this to be true?

For the same volume fraction of particles, larger particles mean large particle spacing. As the particle spacing increases, the shear stress needed to bow dislocations around the obstacles decreases. This can be expressed as:

$$\tau_{BOW} \propto Gb\left(\frac{\sqrt{f}}{R}\right)$$

where G is the shear modulus, b is the Burger's vector, f is the particle volume fraction and R is the particle radius. For big particles to have relatively little effect on yield stress, they must be spaced so far apart that the dislocations can move easily around the particles via bowing; this happens when the particles are much much bigger than the distortion caused by dislocations, i.e. the Burger's vector.

b.) Why could one claim that very small particles/inclusions have relatively little effect on yield stress? What are the conditions that are required for this to be true?

Smaller particles can be susceptible to shearing by the dislocation; i.e., the dislocation cuts through the particle with relatively little resistance and continues to glide, resulting in little or no increase in the stress needed to move dislocations (i.e. the yield stress). This can be expressed as:

$$\tau_{SHEAR} \propto \left(\frac{\gamma_{AFB}}{Gb}\right)\sqrt{\frac{fR}{b}}$$

where γ_{AFB} is the energy associated with the anti-phase boundary between the particle and the matrix; this term can be thought of as the fracture energy of the particle. For particles to shear, they must be relatively weak (such that it is easy to cut them, they have a low anti-phase boundary energy) and comparable to the Burger's vector.

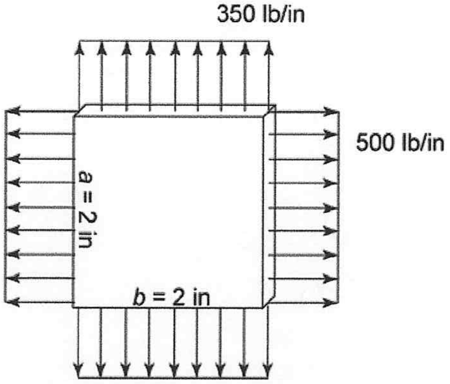
c.) How might one experimentally determine the optimum particle size to maximum yield stress in a precipitate –strengthened alloy?

One can make an alloy with extremely small particles by quenching rapidly from elevated temperatures: this limits the particle size since the atoms comprising the particles do not have time to coalesce into large particles. Then, a number of samples are cut from the alloy, and each sample is heated for increasing intervals, i.e. the samples are subjected to different annealing schedules to grow the particles to different sizes. When the samples are heated, the precipate atoms have sufficiently mobility to grow larger particles (at the expense of smaller particles – the large particles get bigger while the smaller particles get smaller, since larger particles have less surface energy) – this process is known as Oswald ripening. The yield stress for each sample can then be measured and combined with microscopy to determine which particle sizes yield the most significant increases in strength.

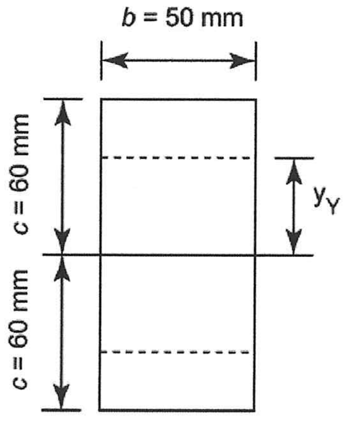
CE 323: Properties and Behavior of Materials **HW #1**
Fall 2003 **Due: Friday, Sept. 12th**

1. A nylon thread is to be subjected to a 2.5 lb tension. Knowing that $E = 0.5x106$ psi, that the maximum allowable normal stress is 6 ksi, and that the length of the thread should not increase by more than 1%, determine the required diameter of the thread.

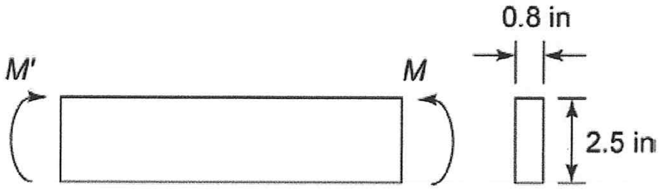
2. A uniform edge load of 500 lb/in and 350 lb/in is applied to the polystyrene specimen. If it is originally square and has dimensions of $a = 2$ in, $b = 2$ in, and a thickness of $t = 0.25$ in, determine its new dimensions a', b', and t' after the load is applied. $E_p = 597\text{x}10^3$ psi, $\nu_p = 0.25$.



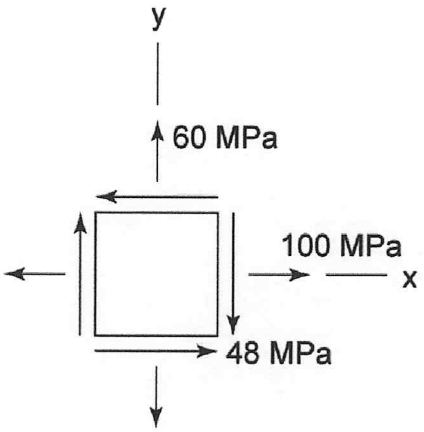
3. A member of uniform rectangular cross section 50 x 120 mm is subjected to a bending moment M = 36.8 kN*m. Assuming that the member is made of an elastic-plastic material with with yield strength of 240 MPa and a modulus of elasticity of 200 GPa, determine the thickness of the elastic core and the radius of curvature of the neutral surface.



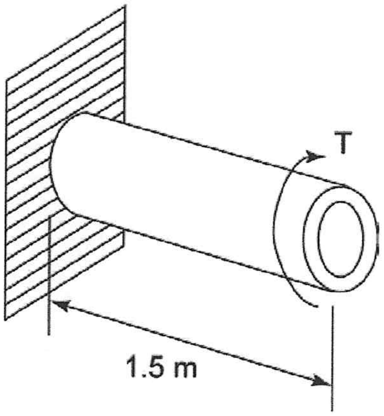
4. A steel bar of 0.8 x 2.5 in. rectangular cross-section is subjected to two equal and opposite couples acting in the vertical plane of symmetry of the bar. Determine the value of the bending moment M which causes the bar to yield. Assume $\sigma_y = 36$ ksi.



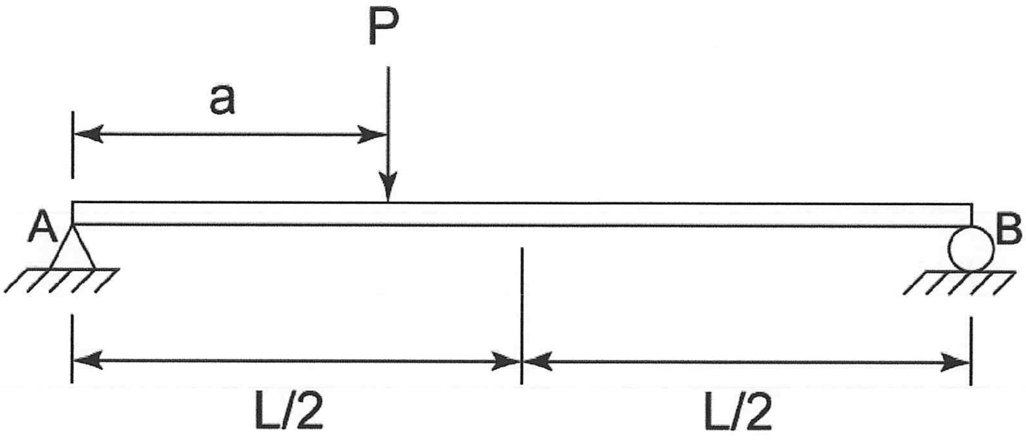
5. For the state of plane stress shown, determine the principal planes and the principal stresses and the stress components exerted on the element obtained by rotating the given element counterclockwise through 30°.



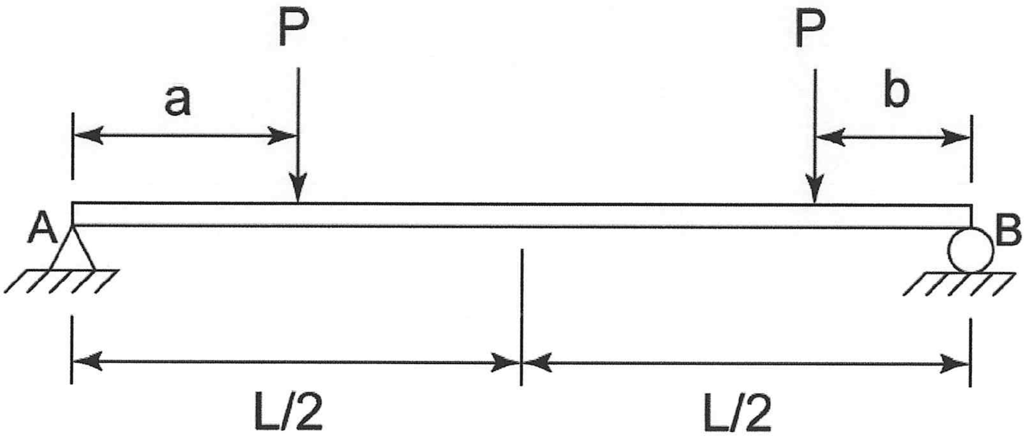
6. A hollow cylindrical shaft is 1.5 m long and has inner and outer diameters respectively equal to 40 and 60 mm. What is the largest torque that may be applied to the shaft if the shearing stress is not to exceed 120 MPa? What is the corresponding minimum value of the shearing stress in the shaft?



7. Determine the deflection at the point of loading ~~and~~(at the mid-span). Assume a = L/2.



8. Determine the deflection at the points of loading and at the mid-span. Assume a= b.



HOMEWORK #1 SOLUTIONS.

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Fall 2003 **Due: Friday, Sept. 12th**

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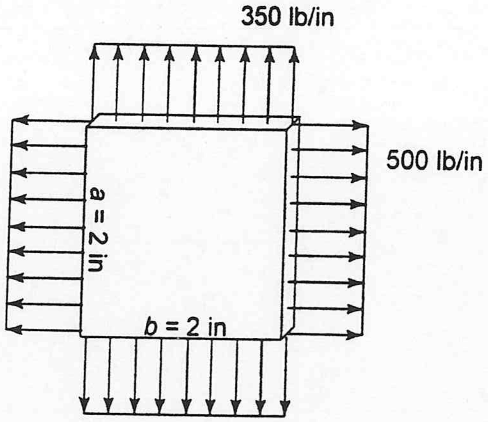

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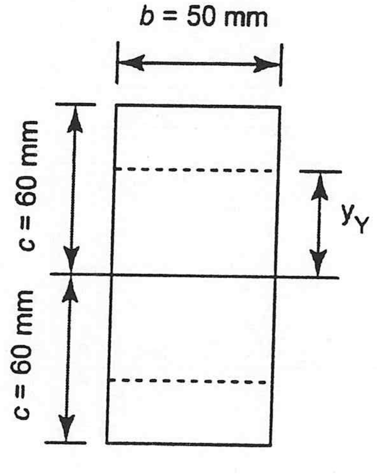
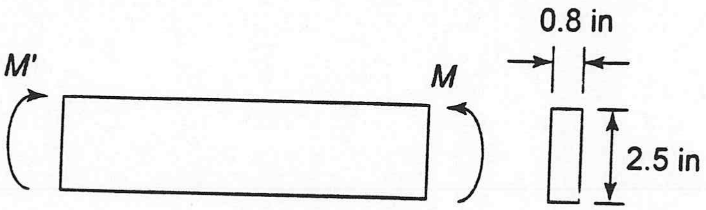
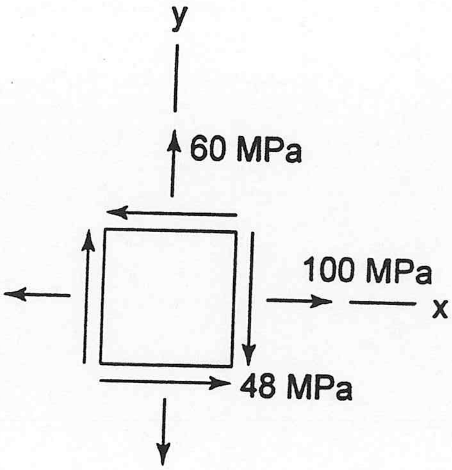


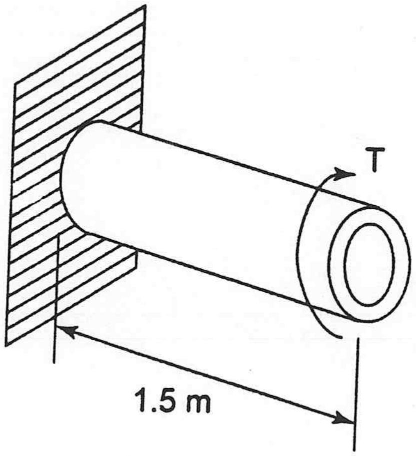
4. A steel bar of 0.8 x 2.5 in. rectangular cross-section is subjected to two equal and opposite couples acting in the vertical plane of symmetry of the bar. Determine the value of the bending moment *M* which causes the bar to yield. Assume $\sigma_y = 36$ ksi.



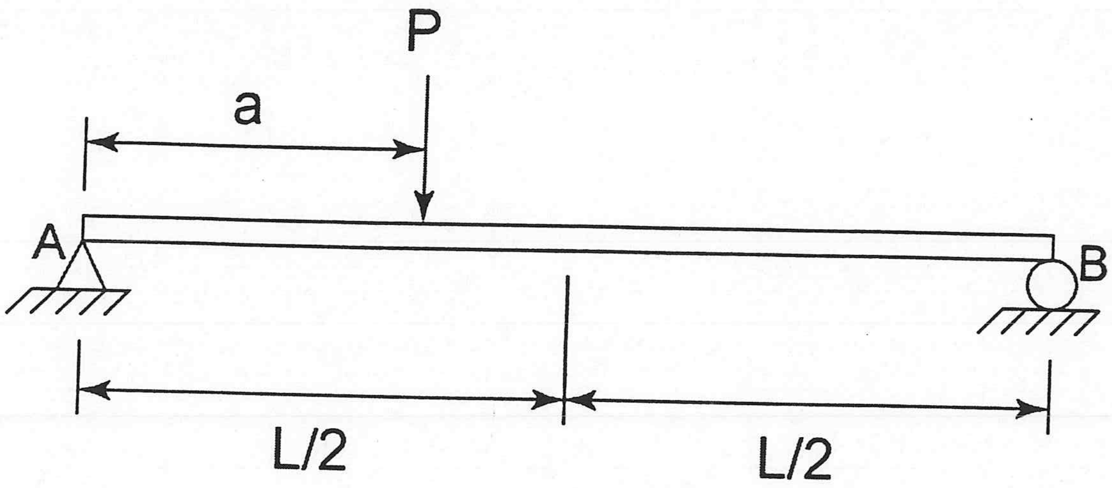
5. For the state of plane stress shown, determine the principal planes and the principal stresses and the stress components exerted on the element obtained by rotating the given element counterclockwise through 30°.



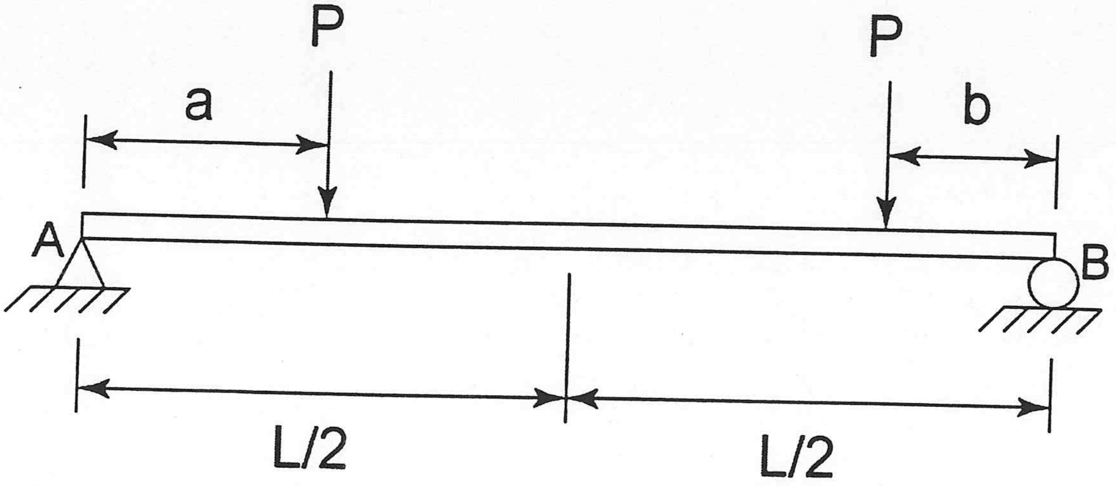
6. A hollow cylindrical shaft is 1.5 m long and has inner and outer diameters respectively equal to 40 and 60 mm. What is the largest torque that may be applied to the shaft if the shearing stress is not to exceed 120 MPa? What is the corresponding minimum value of the shearing stress in the shaft?



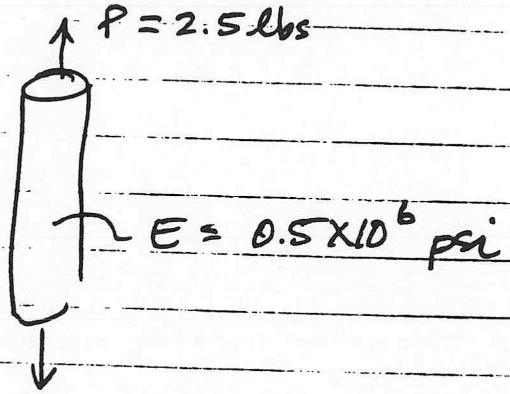
7. Determine the deflection at the point of loading and at the mid-span.



8. Determine the deflection at the points of loading and at the midspan.



①



<u>Stress limit:</u>

$$\sigma_{max} = \frac{P}{A} = \frac{P}{\pi r^2}$$

$$r = \sqrt{\frac{P}{\pi \sigma_{max}}}$$

$$r = 0.0115''$$

<u>Strain limit:</u>

$$\varepsilon_{max} = \frac{\sigma}{E} = \frac{P}{AE}$$

$$r = \sqrt{\frac{P}{\pi E \varepsilon_{max}}} = 0.0126''$$ ← USE THIS: BIGGER DIAMETER MEANS $\sigma < \sigma_{max}$!

②

$$\varepsilon_x = \frac{\sigma_x}{E} - \nu\left(\frac{\sigma_y + \cancel{\sigma_z}^{\,0}}{E}\right)$$

$$= \left(\frac{500 \text{ lb/in}}{0.25 \text{ in}}\right)\cdot\left(\frac{1}{597 \times 10^3 \text{ psi}}\right) - 0.25\left(\frac{350 \text{ lb/in}}{0.25 \text{ in } 597 \times 10^3 \text{ p}}\right.$$

$$= 0.00276$$

$$\varepsilon_y = \frac{\sigma_y}{E} - \nu\left(\frac{\sigma_x + \cancel{\sigma_z}^{\,0}}{E}\right) = \left[\left(\frac{350}{0.25}\right) - 0.25\cdot\left(\frac{500}{0.25}\right)\right] \times \left(\frac{1}{597 \times 10^3 \text{ psi}}\right)$$

$$= 0.00151$$

②

$$\varepsilon_z = \cancelto{0}{\frac{\sigma_z}{E}} - \nu\left(\frac{\sigma_y+\sigma_x}{E}\right) = -0.25\left(\frac{850}{0.25\cdot 597\times10^3}\right)$$

$$= -0.00142$$

$$\Delta_x = b\varepsilon_x \; ; \; b' = b+\Delta_x = (1+\varepsilon_x)b$$

$$\boxed{b' = 2.00552 \text{ in}}$$

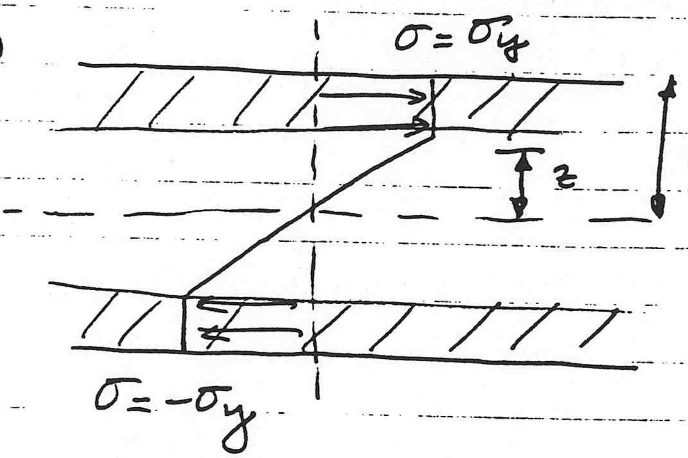
$$\Delta_y = b\varepsilon_y \; ; \; a' = b+\Delta_y = (1+\varepsilon_y)a$$

$$\boxed{a' = 2.00302 \text{ in}}$$

$$\Delta_z = t\varepsilon_z \; ; \; t' = t+\Delta_z = (1+\varepsilon_z)t$$

$$\boxed{t' = 0.24964 \text{ in}}$$

③



$$M = 2b\int_z^c \sigma_y \cdot y\,dy + 2b\int_0^z E\cdot \underbrace{Ky}_{\varepsilon(y)}\cdot y\,dy$$

$$M = b\left[\sigma_y\left(c^2 - z^2\right) + \frac{2}{3} E k z^3\right]$$

But @ $y = z$, $\varepsilon = \varepsilon_y = \frac{\sigma_y}{E} = K \cdot z \Rightarrow K = \frac{\sigma_y}{Ez}$

$$M = \sigma_y b c^2 \left[1 - \frac{1}{3}\bar{z}^2\right] \quad \text{where } \bar{z} \equiv \frac{z}{c}$$

$$36.8 \times 10^3 = \left(240 \times 10^6\right)(0.05)(0.06)^2 \left[1 - \frac{1}{3}\bar{z}^2\right]$$

$$\bar{z} = 0.67 \; : \; \boxed{z \equiv \text{elastic core size} \cong 40 \text{ mm}}$$

Curvature: $K = \frac{\sigma_y}{Ez} = \frac{1}{\rho}$; $\rho = \frac{zE}{\sigma_y}$

$$\boxed{\rho = \frac{(0.04)\left(200 \times 10^9\right)}{\left(240 \times 10^6\right)} \cong 33 \text{ m}}$$

④ At initial yield, $\sigma_{max} = \sigma_y$

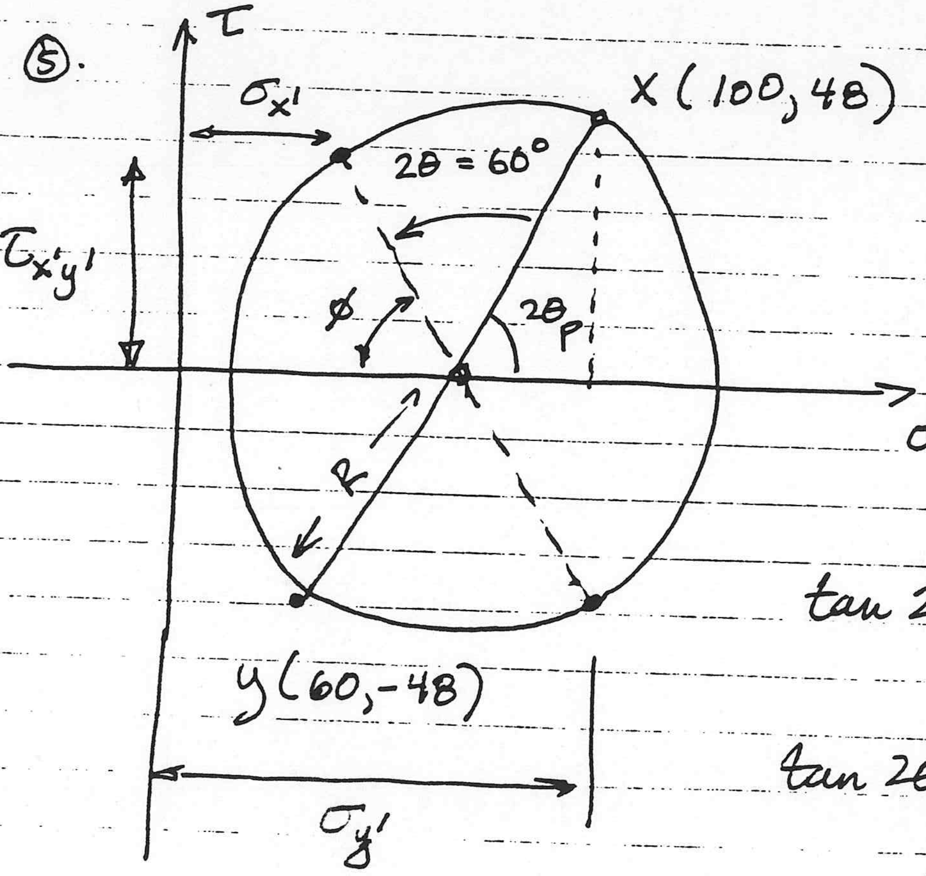
$$\sigma_{max} = \frac{Mc}{I} = \sigma_y \quad ; \quad M = \frac{\sigma_y I}{c}$$

$$M = \frac{\sigma_y\ b \cdot h^3}{12\,(h/2)} = \frac{\sigma_y b h^2}{6}$$

$$M = \left(36 \times 10^3\ \frac{lb}{in^2}\right)\left(\frac{1}{6}\right)\left(0.8\ in\right)\left(2.5\ in\right)^2$$

$$\boxed{M = 3 \times 10^4\ lb\ in\ \ (= 30\ kip \cdot in)}$$

⑤.



$$\sigma_{ave} = \frac{100+60}{2} = 80\ MPa$$

$$\tan 2\theta_p = \frac{\tau_{xy}}{\sigma_x - \sigma_{ave}}$$

$$\tan 2\theta_p = \frac{48}{20}$$

$\theta_p = 33.7°$)

$R = \sqrt{\tau^2 + (\sigma - \sigma\ )^2}$ =

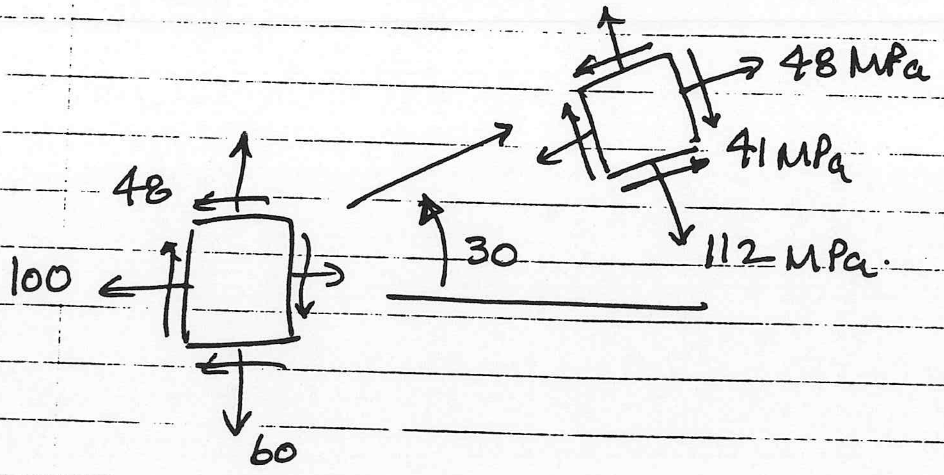
⑤

$$\phi = 180 - 60° - 67.4° = 52.6°$$

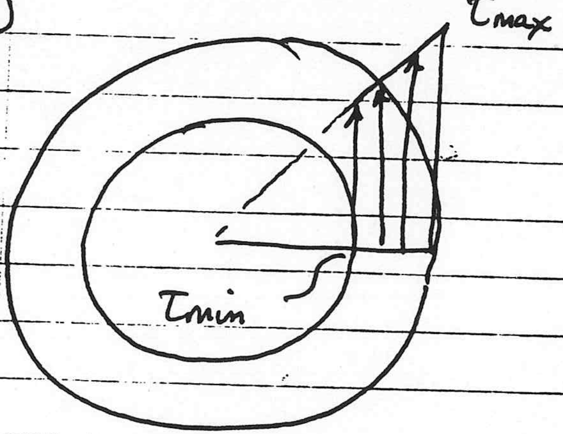
$$\sigma_{x'} = \sigma_{ave} - R\cos\phi = 48.4 \text{ MPa}$$

$$\sigma_{y'} = \sigma_{ave} + R\cos\phi = 111.6 \text{ MPa}$$

$$\tau_{xy'} = R\sin\phi = 41.3 \text{ MPa}$$



⑥



$$\tau = \frac{T \cdot c}{J} = \tau_{max}$$

$$J = J_{out} - J_{in}$$

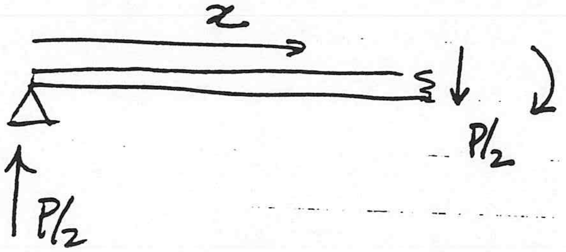
$$= \frac{1}{2}\pi\left(C_o^4 - C_i^4\right) = 1.02 \times 10^{-6} \text{ m}^4$$

$$T_{max} = \frac{J\,\tau_{max}}{c} = \frac{(1.02 \times 10^{-6})(120 \times 10^6)}{0.03} = \underline{\underline{4.08 \times 10^3 \text{ N}\cdot\text{m}}}$$

Geometry (linear stress distribution):

$$\frac{\tau_{out}}{c_{out}} = \frac{\tau_{min}}{c_{in}} \; ; \quad \tau_{min} = \frac{c_{in}}{c_{out}}\tau_{max} = \underline{\underline{80 \text{MPa}}}$$

⑦



$M = \frac{Px}{2}$

$$\frac{d^2v}{dx^2} = \frac{M(x)}{EI} = \frac{Px}{2EI}$$

$$\frac{dv}{dx} = \frac{Px^2}{4EI} + C_1$$

$$v(x) = \frac{Px^3}{12EI} + C_1 x + C_2$$

@ $v(x=0) = 0 : \quad C_2 = 0.$

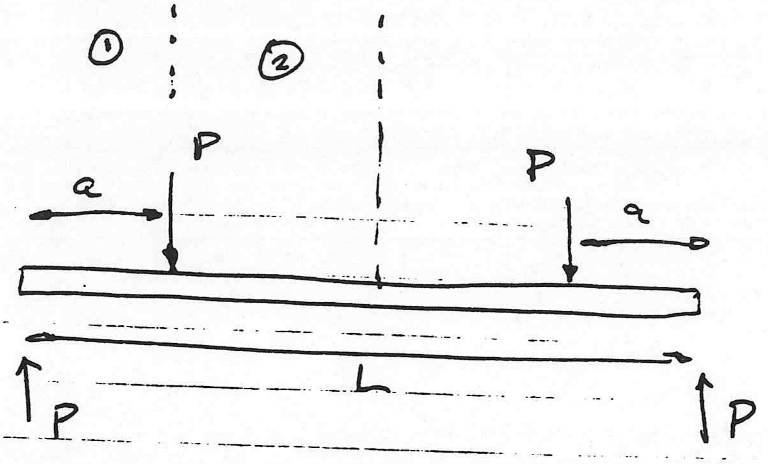
$\frac{dv}{dx}\left(x = \frac{L}{2}\right) = 0$ from symmetry $: \quad C_1 = -\frac{PL^2}{16EI}$

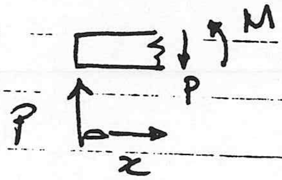
$$v(x) = \frac{Px^3}{12EI} - \frac{PL^2x}{16EI}$$

$$v\left(x = L/2\right) = \delta = \frac{PL^3}{96EI} - \frac{PL^3}{32EI}$$

$$\boxed{\downarrow \delta = \frac{1}{48}\frac{PL^3}{EI}}$$

NOTE: SAME AS CANTILEVER WITH LENGTH 2L AND





For $x < a$: $M = -Px$

$$\frac{d^2 v_1}{dx^2} = \frac{M}{EI} = -\frac{Px}{EI}$$

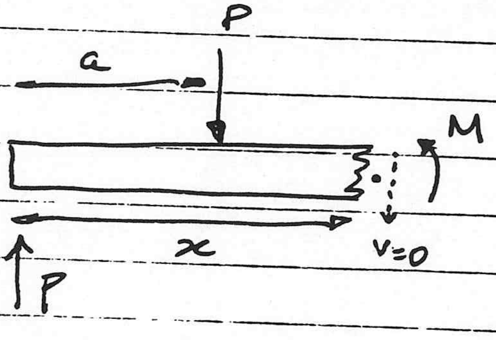
$$\frac{dv_1}{dx} = -\frac{Px^2}{2EI} + C_1$$

$$v_1(x) = -\frac{Px^3}{6EI} + C_1 x + C_2$$

FOR $x > a$ and $x < L/2$:

$$M = P(x-a) - Px$$

$$M = -Pa$$ (const. moment in center span!)



$$\frac{d^2 V_2(x)}{dx^2} = \frac{M}{EI} = -\frac{Pa}{EI} \; ; \; \frac{dV_2}{dx} = -\frac{Pa}{EI}x + C_3$$

$$V_2(x) = -\frac{Pax^2}{2EI} + C_3 x + C_4$$

BOUNDARY CONDITIONS: $V_1(x=0)=0$: $\boxed{C_2 = 0.}$ (i)

(ii) $$\frac{dV_1}{dx} = \frac{dV_2}{dx} \quad @ \ x = a$$

$$-\frac{Pa^2}{2EI} + C_1 = -\frac{Pa^2}{EI} + C_3 \qquad C_1 - C_3 = -\frac{Pa^2}{2EI}$$

(iii) $V_1 = V_2$ @ $x = a$

$$-\frac{Pa^3}{6EI} + C_1 a = -\frac{Pa^3}{2EI} + C_3 a + C_4$$

(iv) $\frac{dV_2}{dx}\left(x=\frac{L}{2}\right) = 0$; $\boxed{C_3 = \frac{PaL}{2EI}}$

From (ii): $\boxed{C_1 = -\frac{Pa^2}{2EI} + \frac{PaL}{2EI}}$

From (iii) $\frac{Pa^3}{3EI} - \frac{Pa^3}{2EI} + \cancel{\frac{Pa^2L}{2EI}} - \cancel{\frac{Pa^2L}{2EI}} = C_4$

$$\boxed{-\frac{Pa^3}{6EI} = C_4}$$

$$\delta_a = v_1(x=a) = -\frac{Pa^3}{6EI} + \frac{Pa^2L}{2EI} - \frac{Pa^3}{3EI}$$

$$= -\frac{Pa^3}{2EI} + \frac{Pa^2L}{2EI} = \boxed{\frac{Pa^2}{2EI}(L-a) = \delta_a}$$

$$\delta_{L/2} = v_2(x=L/2) = -\frac{PaL^2}{8EI} + \frac{PaL^2}{4EI} - \frac{Pa^3}{6EI}$$

$$\boxed{\delta_{L/2} = \frac{PaL^2}{8EI} - \frac{Pa^3}{6EI} =}$$

CE323 Properties and Behavior of Materials **Homework #2**
Fall 2003 **Due: ~~Tuesday~~, Sept. ~~16th~~**

Thursday, Sept. 18th

Problem One:

Mechanical springs function by storing energy via elastic deformation. The strain energy density (i.e. strain energy per unit volume of material) in an axially loaded rod is given by:

$$W = \frac{1}{2}\sigma\varepsilon$$

The "best" material for a spring is one that stores the most elastic energy; this is obviously influenced by the failure stress (strength) of the material, σ_f, and the elastic modulus, E.

1. Derive the performance index for springs, denoted as $M = f(\sigma_f, E)$.
2. Using an appropriate materials map that illustrates combinations of σ_f and E for various materials, identify the class(es) of materials that maximizes the appropriate performance index.
3. Compare the performance indices of brick to silicone and explain any similarities or differences.
4. Briefly discuss factors that are not considered in the performance index and their implications for eliminating classes of materials from the material class(es) identified in part 2.

Problem Two:

Elastic hinges are structures that permit the relative movement of two connect pieces through bending deformation. The ideal spring would not allow permanent deformation (e.g. via plastic flow or failure).

1. Use elementary beam theory to show that the minimum radius of curvature of the bent hinge (i.e. the maximum range of motion) is given by:

$$R = \frac{t}{2}\left(\frac{E}{\sigma_f}\right)$$

2. Use the material map from the first problem to identify the class(es) of materials that make the best springs and explain how you arrived at your conclusion.
3. Briefly comment on the suitability of your choice for two applications: (*i*) the hinge on a shampoo bottle top, and (*ii*) the hinge in a microscale mirror used in an array to do bounce fiber optics signals.

CE323 Properties and Behavior of Materials
Fall 2003

Homework #2
Due: Tuesday, Sept. 16th

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PROBLEM 1: $W = \frac{1}{2}\sigma\varepsilon$ (strain energy density ≡ strain energy per unit volume),

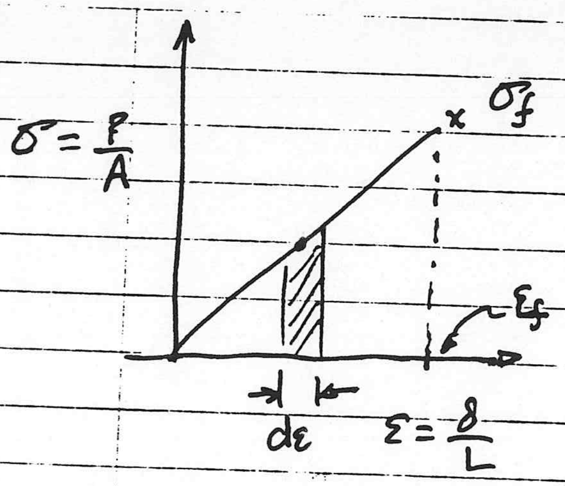
$$W = \frac{1}{2}\sigma \cdot \left(\frac{\sigma}{E}\right)$$

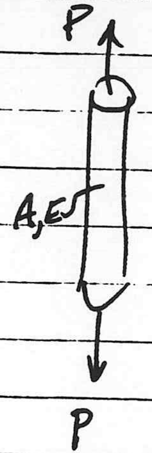
$$= \frac{1}{2}\frac{\sigma^2}{E}$$

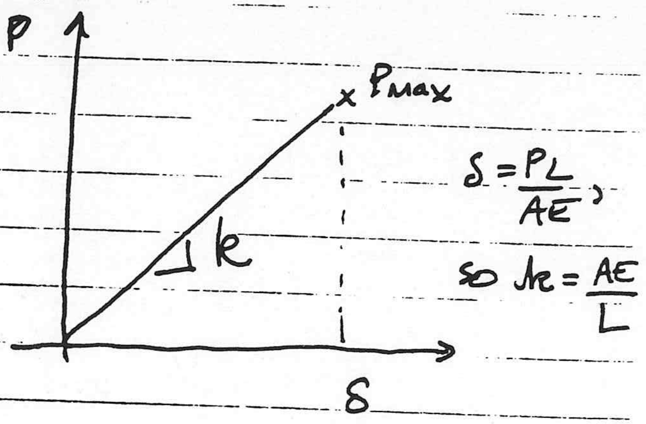
Maximum is stored at failure: $W_{max} = \frac{1}{2}\frac{\sigma_f^2}{E}$

$$\boxed{\text{Performance index, } W_{max} \propto \frac{\sigma_f^2}{E} = M}$$

Note: uniaxial behavior is like a spring!







$$\underset{\text{dens.}}{\text{S.E.}} = \int \sigma\, d\varepsilon \qquad\qquad \text{S.E. total} = \int P\, d\delta$$

$$= \int E\varepsilon\, d\varepsilon \qquad\qquad = \int k\delta \cdot d\delta$$

$$= \frac{1}{2}E\varepsilon^2 \qquad\qquad = \frac{1}{2}k\delta^2$$

$$\leftarrow \delta = \frac{PL}{AE} \rightarrow$$

$$\text{S.E. total} = \frac{1}{2}k\delta^2 = \frac{1}{2}k\left(\frac{PL}{AE}\right)^2 = \frac{1}{2}\frac{AE}{L}\left[\left(\frac{P}{A}\right)\frac{L}{E}\right]^2$$

$$= \frac{1}{2}\frac{AL}{E}\left(\frac{P}{A}\right)^2 = \frac{1}{2}\frac{\sigma^2}{E}\cdot AL$$

$$= \text{Volume}\cdot\frac{1}{2}\sigma\varepsilon \leftarrow \text{volume!}$$

Line A on attached plot: ceramics maximize the performance index. (Draw line with slope = 2 and push right ⇒ Mat'l on far right maximizes σ_f^2/E.)

Line B on attached plot: all materials on this line have the same performance index, so they all 'work' equally well. Brick has same as silicone because its both stronger and stiffer, in the right proportion.

Or conversely, if you want to make a building block out of rubber, it has to be 1600 times bigger than brick!

It all depends on desired stiffness which controls size: energetically, they are equivalent!

$\sigma_f^{RUB} \approx 10$ MPa $\qquad \sigma_f^{BRICK} \approx 400$ MPa $\approx 40\sigma_f^{RUB}$

$E^{RUB} \approx 50$ MPa $\qquad E^{BRICK} \approx 80$ GPa $\approx 1600 E^{RUB}$

$$\frac{\left(\sigma_f^{BRICK}\right)^2}{E^{BRICK}} = \frac{\left(40\,\sigma_f^{RUB}\right)^2}{1600\,E^{RUB}} = \frac{1600}{1600}\frac{\left(\sigma_f^{RUB}\right)^2}{E^{RUB}}$$

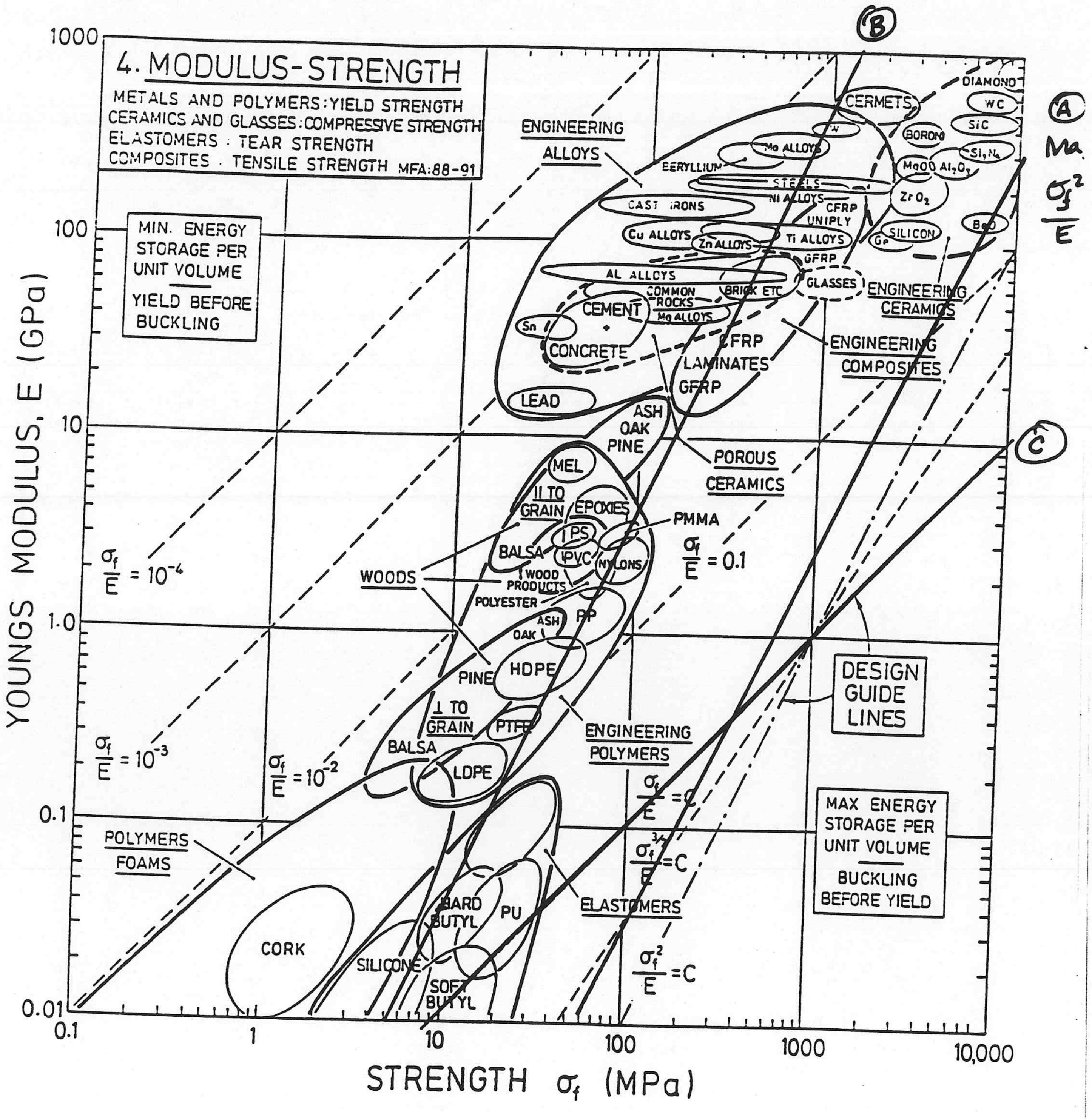
$$= \frac{\sigma_f^{RUB}}{E^{RUB}} \; !$$

Both materials have same performance index!

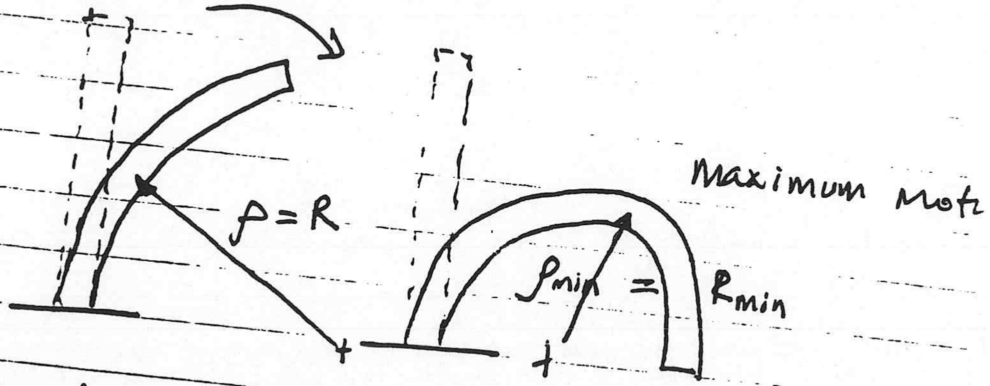
Thus, even though brick is much stronger (40 times), the drastic increase in stiffness (~1600 times) offsets gain by using stronger material. Put another way, both have high performance indices, but for different reasons! Rubber can store a lot of energy by deforming a lot (low modulus), brick stores a lot because its very strong (can take a lot of stress, high E)

The performance index assumes ceramics are loaded in compression! (That's the way table was compiled.) You also want a tough spring — don't want scratch to severely curtail strength. Also, you have to worry about size. If desired spring stiffness is small, an incredibly thin brick is needed to offset large modulus:

Size, $A \propto \dfrac{L \cdot k_{design}}{E}$: This means comparable brick will be 1600 times smaller than rubber (see pg 2)


4. MODULUS-STRENGTH
METALS AND POLYMERS: YIELD STRENGTH
CERAMICS AND GLASSES: COMPRESSIVE STRENGTH
ELASTOMERS : TEAR STRENGTH
COMPOSITES : TENSILE STRENGTH MFA:88-91
MIN. ENERGY STORAGE PER UNIT VOLUME
YIELD BEFORE BUCKLING
MAX ENERGY STORAGE PER UNIT VOLUME
BUCKLING BEFORE YIELD
DESIGN GUIDE LINES
YOUNGS MODULUS, E (GPa)
STRENGTH σf (MPa)
1000
100
10
1.0
0.1
0.01
0.1
1
10
100
1000
10,000
σf/E = 10^-4
σf/E = 10^-3
σf/E = 10^-2
σf/E = 0.1
σf/E = C
σf^3/2/E = C
σf^2/E = C
ENGINEERING ALLOYS
ENGINEERING CERAMICS
ENGINEERING COMPOSITES
ENGINEERING POLYMERS
POROUS CERAMICS
WOODS
ELASTOMERS
POLYMERS FOAMS
DIAMOND
WC
SiC
Si3N4
BORON
MgO
Al2O3
ZrO2
BeO
SILICON
Ge
CERMETS
W
Mo ALLOYS
BERYLLIUM
STEELS
Ni ALLOYS
CFRP UNIPLY
CAST IRONS
Cu ALLOYS
Zn ALLOYS
Ti ALLOYS
GFRP
AL ALLOYS
GLASSES
COMMON ROCKS
BRICK ETC
Mg ALLOYS
CEMENT
CONCRETE
Sn
LEAD
CFRP
LAMINATES
GFRP
ASH OAK PINE
MEL
II TO GRAIN
EPOXIES
PS
PMMA
BALSA
PVC
NYLONS
WOOD PRODUCTS
POLYESTER
PP
ASH
OAK
HDPE
PINE
⊥ TO GRAIN
PTFE
BALSA
LDPE
CORK
HARD BUTYL
PU
SILICONE
SOFT BUTYL
Ⓑ
Ⓐ
Ma
σf²/E
Ⓒ

PROBLEM 2:



Hinge is formed via bending, not moveable components. (Shampoo bottles are an example.)

Maximum motion (top closes!) for minumum radius of curvature. Failure stress dictates how far the hinge can be bent (i.e. it breaks if bent into too small a radius.)

$\sigma_{max} = \sigma_{fail}$ (or σ_f)

$$= E\varepsilon_{max} = E\kappa^{max}\cdot y^{max} = \frac{E y^{max}}{R_{min}} = \frac{Et}{2R_{min}}$$

$$\boxed{R_{min} = \frac{t}{2}\frac{E}{\sigma_f}}$$ ⇒ maximum motion obtained for Rmin (see above); maximize inverse of this

Line C on map: maximize σ_f/E : this is yield strain (want to deform spring as much as possible). Rubbers and polymers have maximum yield strain, $\frac{\sigma_f}{E} \approx 1$! (100% yield strain!)

Hinge on a bottle top: rubber or polymer works fine; position doesn't matter, its either snapped shut or open. Temperature and enviroment not a concern.

Fiber optics mirror: ability to hold a very specific position is everything; small temperature variations change modulus and dimensions of rubber. Very difficult to hold mirror in position as temperature changes. Should use something else and create design that doesn't require large range of motion. Only need to change position a few degrees, so failure strains are not governing factor!

CE323 Properties and Behavior of Materials **Homework #3**
Fall 2003 **Due: Friday, Oct. 3rd**

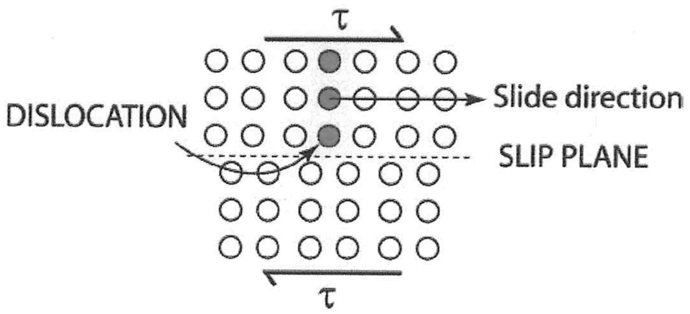
Problem One:

Assume that the theoretical strength of a ceramic is accurately determined by the mechanism model that converts strain energy to surface energy (see class notes or Gordon's text.)

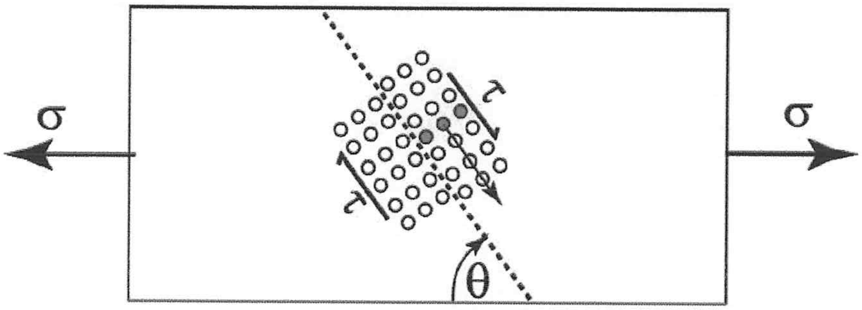
a. Derive an expression for the theoretical strength in terms of the surface energy, the elastic modulus and the atomic spacing. (For consistency, keep the factor of two in front.)
b. Calculate the theoretical strength assuming $\gamma = 1\,J/m^2$, $E = 400\,GPa$ and $a = 3\,\overset{o}{A}$.
c. Calculate the strength (i.e. failure stress) of a plate that has an elliptical hole that is five times wider than it is high.
d. If the strength of a plate with an elliptical hole is 1 GPa, and the width of the ellipse is 1 micron, calculate the tip radius of the ellipse. How sharp is the tip?
e. What is the theoretical strength of the material if an ellipse that is 1 micron wide and has a tip radius of 10 nm leads to a failure stress of 100 GPa?

Problem Two:

Dislocations are defects in crystalline materials; in metals, they are easily moved by application of a shear stress. The plane along which the dislocation moves through the crystal is known as a slip plane. (See figure below.)



Assume that a dislocation will glide along the slip plane when a critical shear stress is reached parallel to the slip plane; call this critical shear stress τ_o.



a. Derive the relationship between the remotely applied tension stress that causes yield (call it σ_y), the critical shear stress and the orientation of the slip plane relative to the direction of loading. (See figure.) Plot the theoretical tensile yield stress as a function of slip plane orientation.

b. What is minimum value of tensile yield stress that is possible, and for which slip plane angles does it occur?

c. Comment on the theoretical tensile yield stress when the slip plane is oriented perpendicular and parallel to the direction of loading; why is are these extreme values never achieved in reality?

CE323 Properties and Behavior of Materials
Fall 2003

Homework #3
Due: Friday, Oct. 3rd

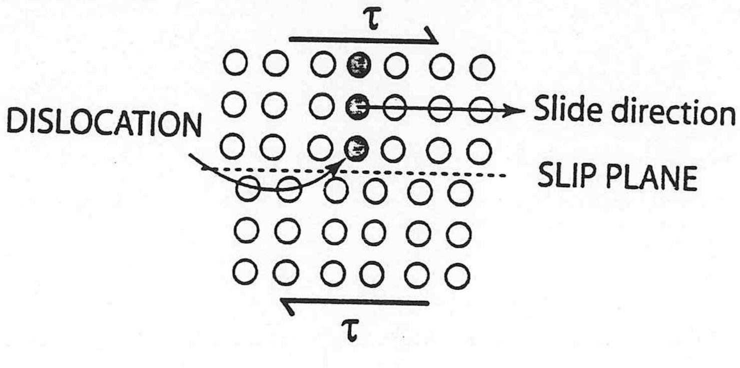
Problem One:

Assume that the theoretical strength of a ceramic is accurately determined by the mechanism model that converts strain energy to surface energy (see class notes or Gordon's text.)

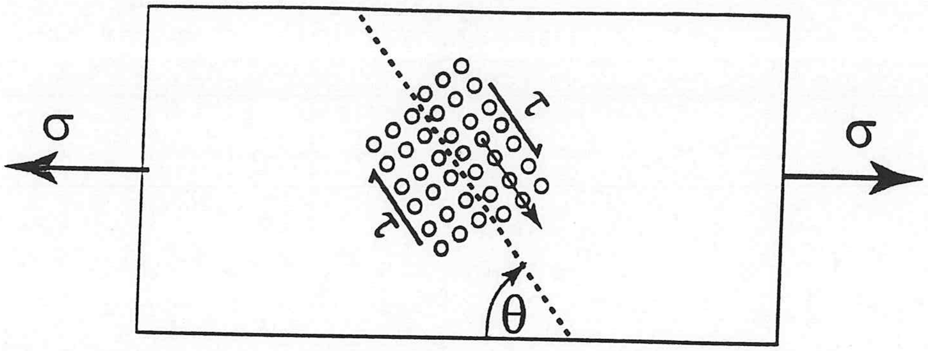
a. Derive an expression for the theoretical strength in terms of the surface energy, the elastic modulus and the atomic spacing. (For consistency, keep the factor of two in front.)
b. Calculate the theoretical strength assuming $\gamma = 1\,J/m^2$, $E = 400\,GPa$ and $a = 3\overset{o}{A}$.
c. Calculate the strength (i.e. failure stress) of a plate that has an elliptical hole that is five times wider than it is high.
d. If the strength of a plate with an elliptical hole is 1 GPa, and the width of the ellipse is 1 micron, calculate the tip radius of the ellipse. How sharp is the tip?
e. What is the theoretical strength of the material if an ellipse that is 1 micron wide and has a tip radius of 10 nm leads to a failure stress of 100 GPa?

Problem Two:

Dislocations are defects in crystalline materials; in metals, they are easily moved by application of a shear stress. The plane along which the dislocation moves through the crystal is known as a slip plane. (See figure below.)



Assume that a dislocation will glide along the slip plane when a critical shear stress is reached parallel to the slip plane; call this critical shear stress τ_c.



a. Derive the relationship between the remotely applied tension stress that causes yield (call it σ_y), the critical shear stress and the orientation of the slip plane relative to the direction of loading. (See figure.) Plot the theoretical tensile yield stress as a function of slip plane orientation.

b. What is minimum value of tensile yield stress that is possible, and for which slip plane angles does it occur?

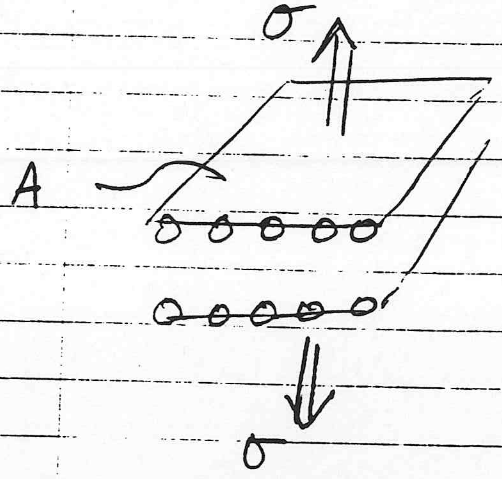
c. Comment on the theoretical tensile yield stress when the slip plane is oriented perpendicular and parallel to the direction of loading; why is are these extreme values never achieved in reality?

Fall 2005

HOMEWORK #3

SOLUTIONS

PROBLEM 1: The theoretical strength can be derived by assuming strain energy stored in stretched lattice is converted to surface energy



Consider two atomic planes spaced distance "a" apart;
A ≡ area of interface
σ ≡ force per unit area on interface.

Strain energy per unit volume created by two planes: $S.E. = \frac{1}{2}\sigma\varepsilon = \frac{1}{2}\frac{\sigma^2}{E}$

Total strain energy ≡ (S.E.) volume $= \frac{1}{2}\frac{\sigma^2}{E} \cdot A \cdot a$

Surface energy: $2 \cdot A \cdot \gamma$, where γ ≡ energy per unit area of surface.

$$\frac{1}{2}\frac{\sigma^2}{E} \cdot A \cdot a = 2A\gamma$$

$$\Rightarrow \boxed{\sigma_c \text{ THEORETICAL STRENGTH} = 2\sqrt{\frac{E\gamma}{a}}}$$

Part b:
$$\sigma_0 = 2\sqrt{\frac{4\times10^{11}\,\frac{N}{m^2}\cdot 1\,\frac{N\cdot m}{m^2}}{3\times10^{-10}\ m}}$$

$$\boxed{\sigma_0 = 73\ GPa}$$

theoretical strength of mat'l
"intrinsic strength"

Part c: Failure occurs when σ_{max} (at ellipse) equals theoretical strength of material

$$\sigma_{max} = \sigma_0 = \sigma_\infty\left(1+2\left(\frac{a}{b}\right)\right)$$

$$\sigma_\infty^{fail} = \frac{\sigma_0}{1+2\left(\frac{a}{b}\right)} = \frac{\sigma_0}{11} \Rightarrow \sigma_\infty^{fail} \approx \boxed{6.64\ GPa.}$$

Thus, hole knocks down strength of plate by about an order of magnitude.

part d: Strength of plate is $\sigma_\infty^{fail} = 1$ GPa.

$$\sigma_{max} = \sigma_0 = \sigma_\infty^{fail}\left(1+2\sqrt{\frac{a}{\rho_{TIP}}}\right)$$

$$\rho_{TIP} = \frac{4a}{\left(\frac{\sigma_0}{\sigma_\infty^{fail}}-1\right)^2} \cong 7.72\times10^{-10}$$

$\cong 8$ Å or about 2-3 atomic spacings.

Implication is that tip radius needed to explain strength is about equal to the atomic spacing $\Rightarrow$ there really isn't a hole there, is there? Also, elasticity theory only appropriate for 10's of atoms:

➤ Stress formula probably not applicable at this scale!

<u>part e.</u> $\sigma_{max} = \sigma_0 = \sigma_\infty^{fail}\left(1 + 2\sqrt{\frac{1\mu m}{10 nm}}\right)$

$$\sigma_0 = 100\,GPa\left(1 + 2\sqrt{100}\right)$$

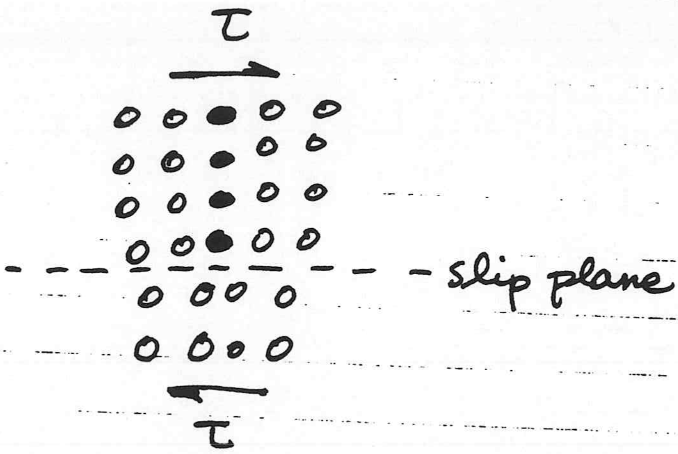
$\sigma_0 = 2{,}100\,GPa$! This means failure strain is $\frac{\sigma_0}{E} \cong 30$ or 3000%! Obviously nonsense, because plate strength (σ_∞^{fail}) is bigger than rough estimate (σ_0), which is too high to begin with! I must have meant:

$\sigma_\infty^{fail} \approx 100\,\underset{x}{\underline{M}}Pa$; then

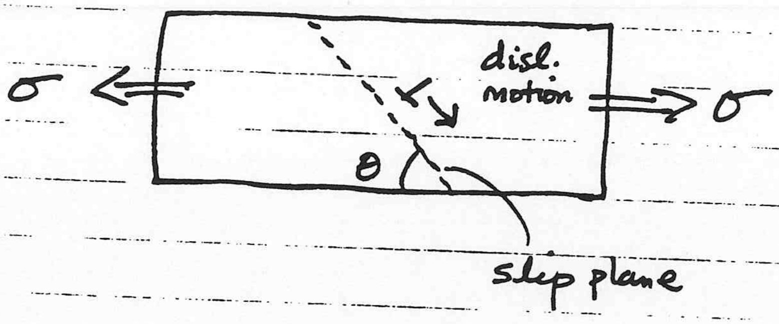
$\sigma_0 \approx 2100\,MPa$ or $2.1\,GPa$, a much more believable result!

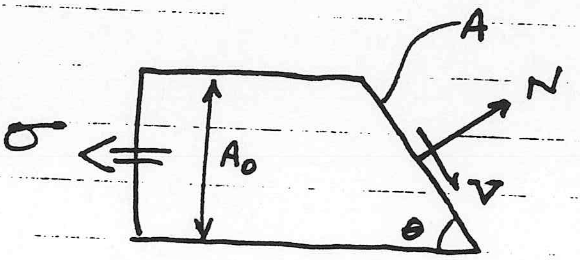
HOMEWORK #3

④



When the shear stress acting on the slip plane hits τ_y, the dislocation moves: call this yielding!





$$\frac{A_0}{A} = \sin\theta$$

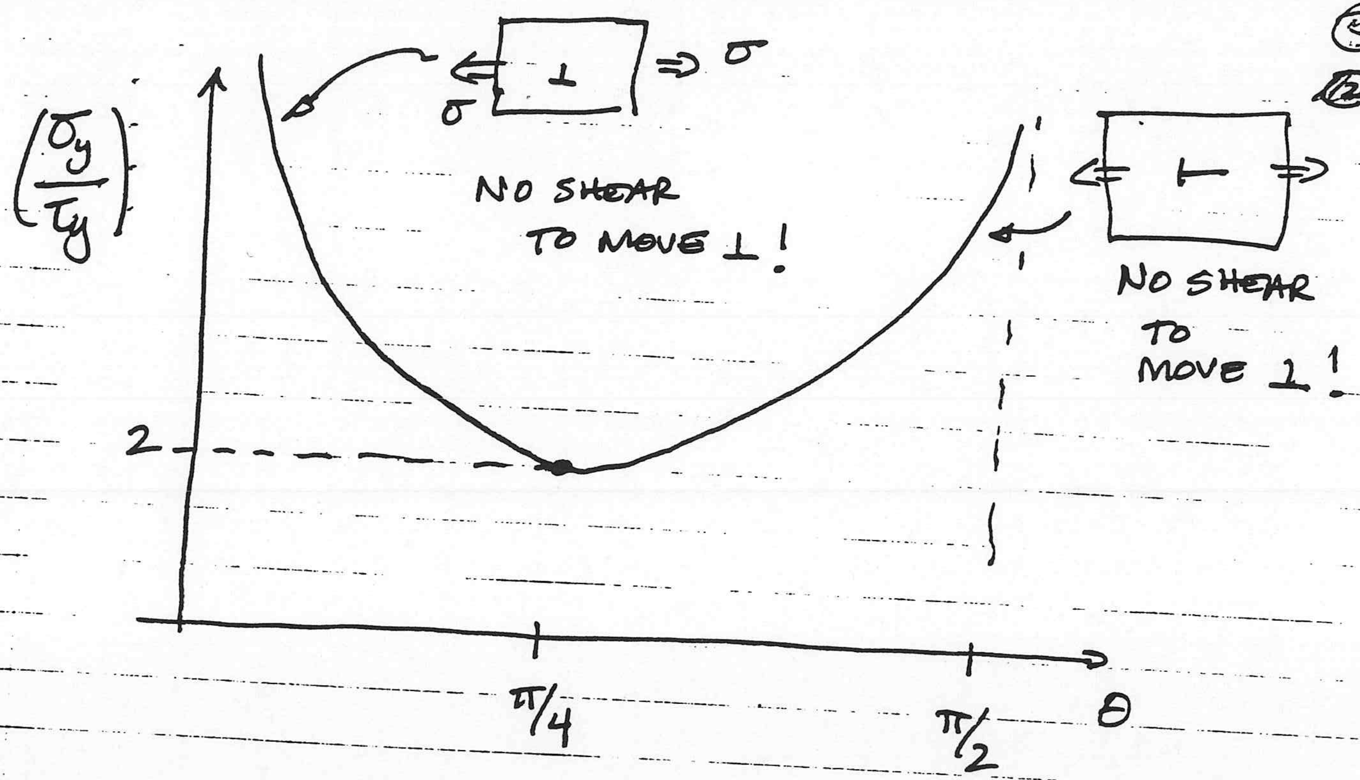
$$\Sigma F_x = \sigma \cdot A_0 = V\cos\theta + N\sin\theta$$

$$\Sigma F_y = 0 = -V\sin\theta + N\cos\theta$$

$$N = V\frac{\sin\theta}{\cos\theta}$$

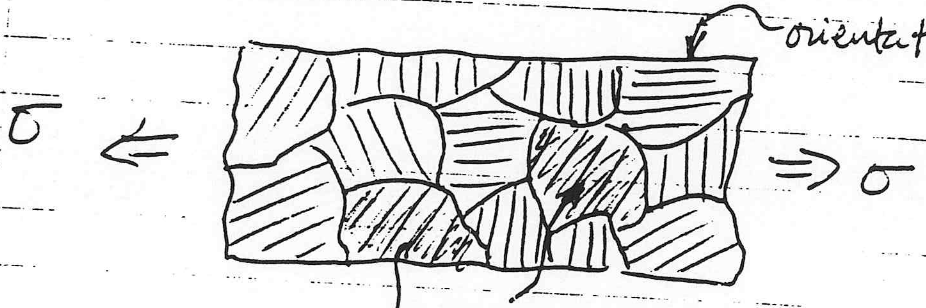
$$\sigma \cdot A_0 = V\left(\cos\theta + \frac{\sin^2\theta}{\cos\theta}\right) = \frac{V}{\cos\theta}$$

$$\sigma_y = \tau_y \cdot \frac{1}{\cos\theta\sin\theta} \quad \text{or} \quad \boxed{\frac{\sigma_y}{\tau_y} = \frac{2}{\sin 2\theta}}$$



If slip plane is aligned parallel to stress axis, or perpendicular, there is no shear acting on the slip plane, no matter how hard I pull on it. Thus, dislocation never moves and yielding doesn't happen!

This doesn't happen in reality because real materials are made up of many crystals with different orientations = these are called grains. Each grain has its own slip planes: one of them is always aligned with plane of maximum shear stress.



orientation of slip planes

"FAVORABLE" ORIENTED GRAINS SLIP FIRST!

Shear activates slip in these crystals first, at $\sigma \approx 2\tau_c$!

CE323 Properties and Behavior of Materials **Homework #6**
Fall 2003 **Due: *BEFORE TURKEY-DAY***

Carbon Steel		Aluminum Alloy	
Grain size (μm)	Yield stress (MPa)	Grain size (μm)	Yield stress (MPa)
406	93	42	233
106	129	16	225
75	145	11	225
43	158	8.5	226
30	189	5.0	231
16	233	3.1	238
1	??	1	??

Problem One:

Plot the yield stress vs. grain size, and show that the data above is consistent with the Hall-Petch relation, given below:

$$\sigma_y = \sigma_o + \frac{k_y}{\sqrt{d}}$$

Determine the parameters σ_o and k_y. Micro-alloyed steels contain small additions of vanadium or niobium that permit smaller grain sizes. Similarly, advanced aluminum alloys containing special types of particles can produce smaller grain sizes. Compare the benefit achieved for each class of material if the grain size is reduced from 150 μm to 1 μm.

Problem Two:

An aluminum alloy has coherent particles whose growth rate is described by:

$$\frac{dV}{dt} = \frac{C}{T}\exp\left(\frac{-Q}{RT}\right) = \text{constant, at constant temperature}$$

where V is the average particle volume, t is the time that the alloy has been annealed at temperature $T = 473$ Kelvin, R is the gas constant, $Q = 130$ kJ/mol is activation energy for diffusion of particle atoms, and C is constant equal to $4\text{x}10^{-11}$ m^3 K/sec.

The shear stress that will bow dislocations around the particles is given by:

$$\tau_{BOW}(t) \approx Gb\frac{\sqrt{f}}{R(t)}$$

where G is the shear modulus of aluminum, R is the particle radius, f is the volume fraction of particles, and b is a measure of the lattice distortion due to a dislocation (define as the Burger's vector).

The shear stress needed to shear the particles is given by:

$$\tau_{SHEAR}(t) \approx G\left(\frac{\gamma_{AFB}}{Gb}\right)^{3/2}\left(\frac{R(t)f}{b}\right)^{1/2}$$

Assume that the particles start with an initial diameter of 10 nm, the anti-phase boundary energy is $\gamma_{AFB} = 0.03$ J/m^2, the volume fraction of particles is $f = 3\%$, and $b = 0.5$ nm.

Calculate and plot the shear yield strength of the material as a function of annealing time from 1 to 1000 hrs. Note that the mechanism that occurs at lowest stress is the one that takes place: e.g., the yield strength will be equal to that needed to bow dislocations if that stress is lower than that needed to shear precipitates.

Determine the annealing time that produces the largest shear strength.

CE323 Properties and Behavior of Materials **Homework #6**
Fall 2003 **Due:** ***BEFORE TURKEY-DAY***

SOLUTIONS

Carbon Steel		**Aluminum Alloy**	
Grain size (μm)	Yield stress (MPa)	Grain size (μm)	Yield stress (MPa)
406	93	42	*223 (note typo fixed)*
106	129	16	225
75	145	11	225
43	158	8.5	226
30	189	5.0	231
16	233	3.1	238
1	**750**	1	**241**

Problem One:

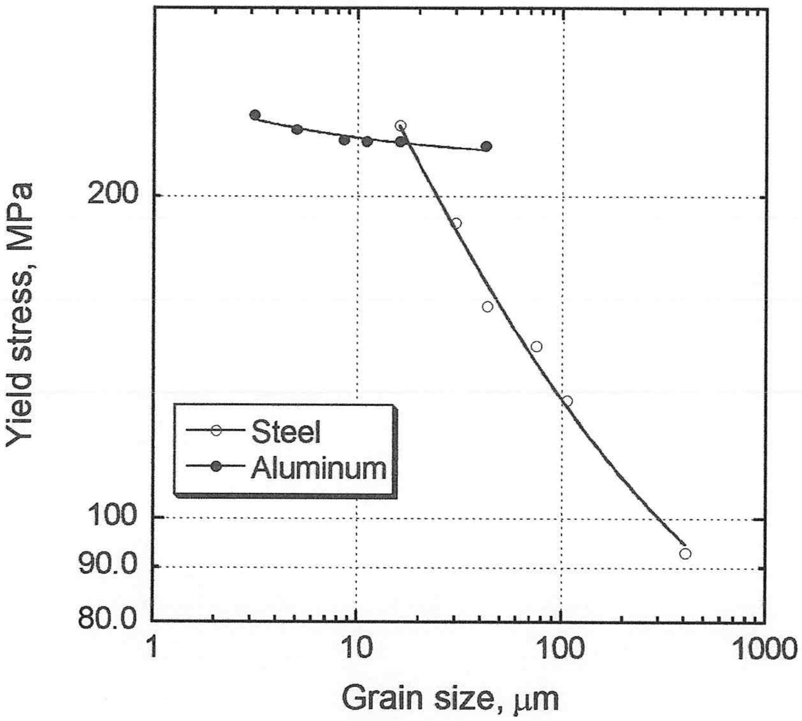
The yield stress vs. grain size is plotted on the next page, and indeed shows that the data above is extremely consistent with the Hall-Petch relation, given below:

$$\sigma_y = \sigma_o + \frac{k_y}{\sqrt{d}}$$

The fitting parameters are listed in the figure. It is clear that a dramatic increase in strength is predicted for the carbon steel when the grain size is reduced to one micron, while relatively little increase is obtained for the aluminum alloy. Using the above formula and the fitting parameters in the figure, one obtains a factor of ten increase in yield strength from the large grain value for steel, while the perctenage increase for aluminum is of the order of 5%. This supports the data reported in class, which was $k_y \sim 70MN\sqrt{\mu m}$ for FCC metals like aluminum, and $k_y \sim 400MN\sqrt{\mu m}$ for BCC metals like iron-based steel. Thus, BCC alloys are much more sensitive to grain size than FCC metals.

σ_o (steel) = 60.5 MPa, k_y = 689 MPa $(\mu m)^{0.5}$

σ_o (alum) = 215.5 MPa, k_y = 36 MPa $(\mu m)^{0.5}$



Problem Two:

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where V is the average particle volume, t is the time that the alloy has been annealed at temperature $T = 473$ Kelvin, R is the gas constant, $Q = 130$ kJ/mol is activation energy for diffusion of particle atoms, and C is constant equal to 4x10^{-11} m^3 K/sec.

The shear stress that will bow dislocations around the particles is given by:

$$\tau_{BOW}(t) \approx 0.5Gb\frac{\sqrt{f}}{R(t)} \quad \textbf{\textit{(note new pre-factor)}}$$

where G is the shear modulus of aluminum, R is the particle radius, f is the volume fraction of particles, and b is a measure of the lattice distortion due to a dislocation (define as the Burger's vector).

The shear stress needed to shear the particles is given by:

$$\tau_{SHEAR}(t) \approx 3G\left(\frac{\gamma_{AFB}}{Gb}\right)^{3/2}\left(\frac{R(t)f}{b}\right)^{1/2}$$ ***(note new pre-factor)***

Assume that the particles start with an initial diameter of 10 nm, the anti-phase boundary energy is $\gamma_{AFB} = 0.03$ J/m^2, the volume fraction of particles is $f = 3\%$, and $\boldsymbol{b} = \mathbf{0.4}$ **nm.** ***(Note new improved Burger's vector).***

Calculate and plot the shear yield strength of the material as a function of annealing time from 1 to 1000 hrs. Note that the mechanism that occurs at lowest stress is the one that takes place: e.g., the yield strength will be equal to that needed to bow dislocations if that stress is lower than that needed to shear precipitates.

Determine the annealing time that produces the largest shear strength.

$$\frac{dV}{dt} = \frac{C}{T}\exp\left(\frac{-Q}{RT}\right) = 3.52x10^{-28}\left(\frac{m^3}{\text{sec}}\right)$$

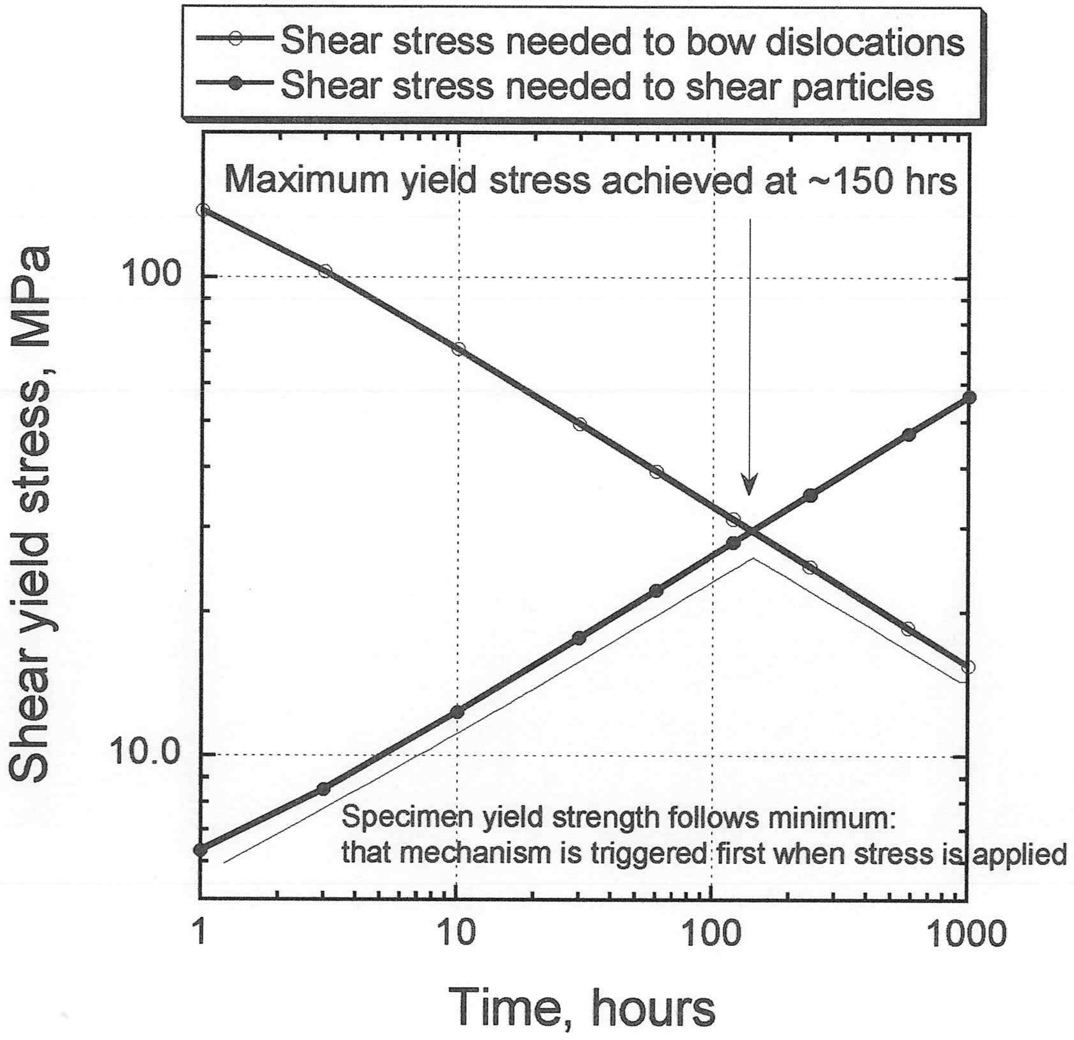
$$V(t) = V(t=0) + 3.52x10^{-28}t \ \left(m^3\right)$$

$$V(t=0) = \frac{4\pi}{3}R^3 = \frac{4\pi}{3}\left(5x10^{-9}m\right)^3 = 5.24x10^{-25}\,m^3$$

$$R(t) = \left(\frac{3}{4\pi}V(t)\right)^{1/3} = \left(\frac{3}{4\pi}\left[5.24x10^{-25} + 3.52x10^{-28}t\right]\right)^{1/3} m$$

Time (hrs)	Time (secs)	Radius (m)	Shear(bow), N/(m*m)	Shear(shear), N/(m*m)
1	3600	7.53388E-09	1.379E+08	6.357E+06
3	10800	1.01077E-08	1.028E+08	8.528E+06
10	36000	1.46595E-08	7.089E+07	1.237E+07
30	108000	2.09544E-08	4.959E+07	1.768E+07
60	216000	2.63409E-08	3.945E+07	2.223E+07
120	432000	3.31495E-08	3.135E+07	2.797E+07
240	864000	4.17419E-08	2.490E+07	3.522E+07
580	2088000	5.5997E-08	1.856E+07	4.725E+07
1000	3600000	6.71396E-08	1.548E+07	5.665E+07

NOTE: I used slightly different constants that yield more accurate results – recall the e-mail that I sent regarding the pre-factors. I also reduced the Burger's vector value, as I had a different number in my notes. This shifts the ideal annealing time to smaller values (on the order of 4 days). You get essentially the same results with the numbers that I gave originally, but obtain drastically larger anneal times that fall outside the 1000 hr window. (Note that 1000 hrs ~ 40 days!). Assignments with proper use of the constants I originally gave get full credit.



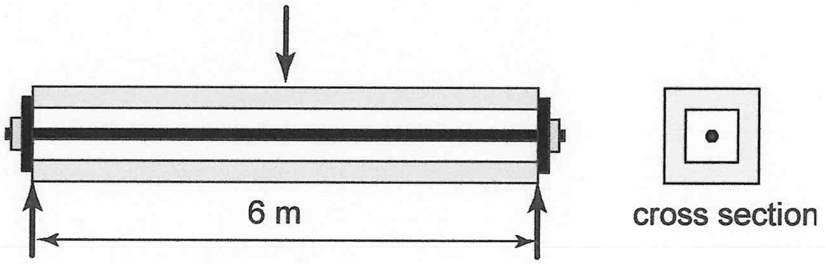
CE323 Properties and Behavior of Materials **Homework #7**
Fall 2003 **Due:** ***12/5/03***

Problem One:

Determine the components of an appropriate mix for a concrete footing that is approximately 12"x12"x12", and requires a compressive strength of 3 ksi. Assume that the mix will involve 2.5% entrained air, and that the aggregate and sand available have 2% adsorbed water. (Hint: the one-page hand-out in class that has data on concrete can be used to solve for acceptable water-to-cement ratios, aggregate volume per cubic yard, etc.)

Problem Two:

A beam is made of a concrete hollow square section that is loaded in compression using a steel reinforcing rod that is threaded on the ends, as shown below. The beam is then subjected to three point bending.



Concrete: outside = 30 cm, inside = 20 cm
Steel shaft diam. = 2.0 cm

The stress intensity factor for a shallow surface crack is approximately $K = 1.12\sigma\sqrt{\pi a}$. The toughness of the concrete is $K_{Ic} = 0.5 MPa\sqrt{m}$, and its compressive strength is $\sigma_c = 12$ MPa. The aggregate size is 1 cm.

a.) Solve for the maximum load at the center of the beam that can be supported if there is no pre-tension, assuming a flaw size of 0.5 cm.
b.) Solve for the compressive stress needed to suppress crack growth if the flaw size is 0.1 cm, and the center load is P = 20 kN
c.) Solve for the maximum flaw size that can be tolerated, assuming the load is 15 kN and the steel rod does not fail. What are the maximum/minimum stresses in each component?
d.) What is the qualitative load-deflection behavior of the structure if the concrete yields in compression (and does not fail in tension)?

CE323 Properties and Behavior of Materials
Fall 2003

Homework #7
Due: ***12/5/03***

SOLUTIONS

Problem One:

Determine the components of an appropriate mix for a concrete footing that is approximately 12"x12"x12", and requires a compressive strength of 3 ksi. Assume that the mix will involve 2.5% entrained air, and that the aggregate and sand available have 2% adsorbed water. (Hint: the one-page hand-out in class that has data on concrete can be used to solve for acceptable water-to-cement ratios, aggregate volume per cubic yard, etc.)

Aggregate and slump choice:

The aggregate size should be no larger than 20% of the dimension of the form, which is ~2.4". I choose an aggregate size of 2". Such a small concrete footing doesn't seem to difficult to fill, so I can choose a relatively small slump since workability isn't a real concern.

Water content:

If I choose 2" aggregate and a slump of 1", then the approximate water content per cubic yard is ~250 lbs/yd^3, according to Figure 17-10 in the handout.

Water-to-cement ratio and cement content:

The required strength sets the water to cement ratio. From Figure 17-9 for samples with entrained air, the water-to-cement ratio that yields a compressive strength of 3 ksi is ~0.6. The cement content is then:

$$lbs\text{ cement/yd}^3 = \frac{H_2 0lbs}{yd^3} \cdot \frac{lbs\text{ cement}}{H_2 0lbs} = \frac{H_2 0lbs}{yd^3} \cdot \frac{1}{\text{wc ratio}} = \frac{250lbs}{0.6} = 417lbs\text{ cement}$$

Aggregate content:

The volume ratio of aggregate (volume of aggregate per cubic yard of concrete) can be estimated using Figure 17-11 in the handout. Assuming that we use sand that is between coarse and fine, our aggregate size indicates a volume ratio of aggregate of ~0.73. This is based on the bulk density of aggregate: the true, actual volume occupied by the aggregate is:

$$\textit{Actual}\text{ volume of aggregate} = \text{bulk volume} \cdot \frac{\text{true volume}}{\text{bulk volume}}$$

$$= \text{bulk volume} \cdot \frac{\text{true density}}{\text{bulk density}} = 0.73 \cdot 0.6 \sim 0.44 \frac{\text{cubic yards aggregate}}{\text{cubic yard concrete}}$$

Volumes:

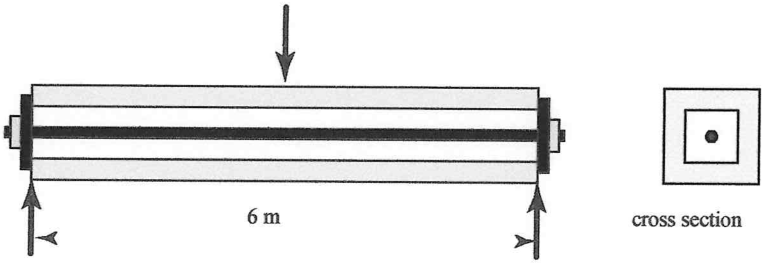
Water: 250 lbs (ft^3/62.4 lbs) = 4 ft^3
Cement: 417 lbs (ft^3/190 lbs) = 2.19 ft^3
Aggregate: 0.44 yds^3 (27 ft^3/yd^3) = 11.88 ft^3
Air: 0.025% (27 ft^3/yd^3) = 0.675 ft^3
Sand: 27 – 4 – 2.19 – 11.88 – 0.675 ft^3 = 8.34 ft^3

Measured units for a cubic yard:

Aggregate: 11.88 ft^3 (170 lbs/ft^3) * 1.02 = 2060 lbs (weight of water adds to meas. wt.)
Sand: 8.34 ft^3 (160 lbs/ft^3) * 1.02 = 1360 lbs
Cement: 417 lbs (sack/94 lbs) = 4.5 sacks
Water: 250 lbs – 0.02*(2060+1360 lbs) = 182 lbs
= 182 lbs (ft^3/62.4 lbs) (7.48 gals/ft^3) = 22 gals

Problem Two:

A beam is made of a concrete hollow square section that is loaded in compression using a steel reinforcing rod that is threaded on the ends, as shown below. The beam is then subjected to three point bending.



Concrete: outside = 30 cm, inside = 20 cm
Steel shaft diam. = 2.0 cm

The stress intensity factor for a shallow surface crack is approximately $K = 1.12\sigma\sqrt{\pi a}$. The toughness of the concrete is $K_{Ic} = 0.5 MPa\sqrt{m}$, and its compressive strength is $\sigma_c = 12\text{ MPa}$. The aggregate size is 1 cm.

a.) Solve for the maximum load at the center of the beam that can be supported if there is no pre-tension, assuming a flaw size of 0.5 cm.

The crack is on the bottom of the beam, the tension side. The maximum stress occurs at the location with the maximum moment, which is directly under the center load:

$$\sigma = \frac{Mc}{I} = \frac{Plc}{4I}$$

where l is the length of the beam, and c is the distance from the neutral axis to the outermost fiber in the beam. The maximum load is determined by setting the stress intensity factor of the crack equal to the toughness:

$$K = K_{Ic} = 1.12\sigma\sqrt{\pi a} = 1.12\frac{Plc}{4I}\sqrt{\pi a}$$

Here are the numbers:

$$I = \left(\frac{bh^3}{12}\right)_{outer} - \left(\frac{bh^3}{12}\right)_{inner} = \left(\frac{0.3^4 m^4}{12}\right) - \left(\frac{0.2^4 m^4}{12}\right) = 5.417x10^{-4} m^4$$

$$P = \frac{4K_{Ic}I}{1.12lc\sqrt{\pi a}} = \frac{4 \cdot 0.5x10^6 Pa\sqrt{m} \cdot 5.417x10^{-4} m^4}{1.12 \cdot 6m \cdot 0.15m \cdot \sqrt{\pi 0.005m}} = 8.6kN$$

b.) Solve for the compressive stress needed to suppress crack growth if the flaw size is 0.1 cm, and the center load is P = 20 kN.

The maximum tensile stress that can be tolerated near the crack is:

$$\sigma_{\max} = \frac{K_{Ic}}{1.12\sqrt{\pi a}} = \frac{0.5x10^6 Pa\sqrt{m}}{1.12\sqrt{\pi 0.001m}} \sim 8MPa$$

The bending stress due to the center load is:

$$\sigma = \frac{Plc}{4I} = \frac{20x10^3 N \cdot 6m \cdot 0.15m}{4 \cdot 5.417x10^{-4} m^4} = 8.31MPa$$

Therefore, the compressive stress needed to reduce the tension due to bending to an acceptable level is:

$\sigma_c = \sigma_{\max,allow} - \sigma_{bending} = -0.31MPa$ (negative indicates compression) c.) *Solve for the maximum flaw size that can be tolerated, assuming the load is 15 kN and the steel rod does not fail. What are the maximum/minimum stresses in each component?*

The maximum flaw size is achieved when the tensile stress on the cracked face is a minimum. The tensile stress on the crack face is minimized when the stress on the compressive face is just at compressive yield. The compressive stress that is applied in the axial direction is then:

$\sigma_c = \sigma_{\max,allow} - \sigma_{bending} = -12MPa - (-6.23MPa) = -5.77MPa$.

This means that the stress on the tensile side is:

$$\sigma = \sigma_{bending} + \sigma_c = 6.23MPa - 5.77MPa = 0.46MPa$$

The flaw size that can be tolerated when subjected to this stress is:

$$a = \frac{1}{\pi}\left(\frac{K_{Ic}}{1.12\sigma}\right)^2 = \frac{1}{\pi}\left(\frac{0.5x10^6\,Pa\sqrt{m}}{1.12\cdot 0.46x10^6\,m}\right)^2 = 30cm$$

This flaw size is actually larger than the tensile side of the member!!! Hence, if you add so much axial compression (as tolerated by the large compressive strength of the concrete), you completely suppress crack growth – the entire bottom half of the structure can be cracked, but the compression is large enough to close the crack tip (not necessarily the entire crack faces though).

The stress in the steel rod is tensile to counteract the compression placed on the concrete; picture tightening a bolt to squeeze the concrete – the bolt goes into tension. Since the steel rod lies on the neutral axis and is relatively small, bending stresses can be reasonably neglected (as they were in the application of the bending equations used above). Noting that bending stresses do not contribute to sum of axial forces, the stresses are related by:

$\sum F_x = 0 = \sigma_{concrete}\cdot A_{concrete} + \sigma_{steel}\cdot A_{steel} = 0$, where stresses are uniform across the cross section.

$$\sigma_{steel} = -\frac{A_{concrete}}{A_{steel}}\sigma_c = -\frac{0.3^2 - 0.2^2\,m^2}{\pi(0.02)^2\,m^2}(-12MPa) = 477MPa \text{ (tension!)}$$

This is a fairly large stress that would make the reinforcement susceptible to cracking induced by corrosion or damage during assembly.

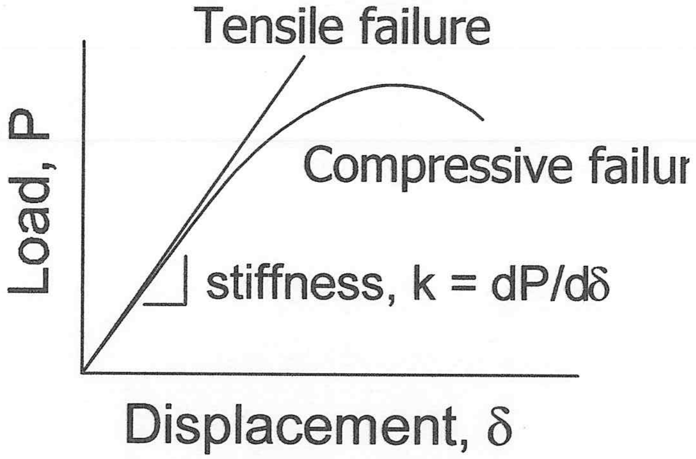
In summary, the stresses in each member are:

Concrete: Maximum (tensile): 0.46 MPa, Minimum (compressive): 12 MPa.

Steel: uniform -> maximum = minimum: 477 MPa.

d.) *What is the qualitative load-deflection behavior of the structure if the concrete yields in compression (and does not fail in tension)?*

As the compressive side begins to crush, it can still carry some load, as indicated by the stress-strain curves for concrete handed out and discussed in class. Unlike a crack that would snap the structure into two pieces, compressive failure appears as an initially gradual decrease in the stiffness of the structure. This is illustrated below.



CE363 Materials Laboratory **Experiment 1**
Fall 2003 **Due, September 23rd**

This laboratory assignment is a thought experiment intended to get you thinking about the challenges of designing a materials characterization experiment. The task is simple to define, but perhaps not so simple to achieve (or is it?). You may work in groups of up to four (or fly solo for the academic glory) and turn in a single lab report.

Develop a method to measure the elastic properties of gelatin, most commonly known commercially as Jell-O. While ultimately both the elastic modulus and Poisson's ratio are desired, focus mainly on modulus. (Poisson's ratio is usually far more challenging.) The basic questions you need to address are:

- What shape will your specimen be?
- How will force be measured?
- How will displacements be measured?
- How are stress and strain related to the specimen geometry, forces and displacements?

Your response will be graded on the following criteria:

- *Will it work*? 50 pts. (As determined by a panel of materials testing experts. Issues like whether or not you get a single data point or multiple data points will play a role.)
- *Is it feasible*? 40 pts. (Many great ideas would work but will be nearly impossible to implement. Cleverness and simplicity will be rewarded handsomely.)
- *How hard do I have to work to interpret your idea*? 10 pts. (This corresponds to clarity of presentation – make sure I can quickly grasp your ideas and the important details that will determine if your method will work and be feasible. A brief 1-3 page typed report with schematic or conceptual drawings would work nicely.)

To create some real world motivation (aside from your grade), the three or four most attractive methods will be considered for implementation into an actual experiment. If feasible, it will be an additional lab in CE363, with the winning authors exempt from having to turn in a lab report. Your names will also be forever credited with the design of the experiment: you'll be famous among future generations of UVa engineers. (Moreover, truly exceptional and original ideas would be publishable, as gelatin has comparable properties to biological materials that are a hot research area.) This introduces a design constraint: *instrumentation must cost less than $2,000*. So, no atomic force microscopes or nanoindentation systems, please! (The K.I.S.S. approach is advisable.) ***Also, you must pledge this: no internet assistance, aside from determining cost.***

Your ace-in-the-hole: you can obviously form whatever specimen geometry you wish; this greatly facilitates design. For example, you can design a specimen with a different shape where the specimen is to be gripped.

Some things to consider: Tension? Compression? Torsion? Bending? Dead weight loading? Levers to amplify displacements to make them easier to observe?

Coming next...measuring the toughness of cheese (actually easier, I think)...

CE 363 Materials Laboratory #1
Elastic Properties of Gelatin
September 23, 2003

Beth Abel
Catherine Hovell
Michelle Smith

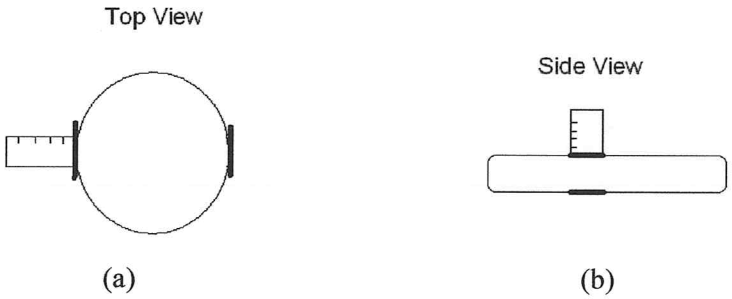
The elastic modulus of gelatin (more commonly known as Jell-O) is important in the scientific realm because of its applications in biotechnology. The material properties of gelatin tend to relate closely to the properties of human tissue. Determining the modulus and the Poisson's ratio of gelatin gives a good estimate for these same properties of human tissue and can thus be used in research in this field.

The geometry of the Jell-O mold in this experiment will be similar to a Petri dish. In fact, the Jell-O could be prepared in a Petri dish. The goal is to create a thin, circular disk of Jell-O with a radius much larger than the thickness. In order to verify the reliability of the test method, various radii and thickness combinations could, and should, be tested. If the thickness gets too great, the Jell-O will likely not deflect on the top and bottom, where it is in contact with other surfaces, but rather bow out in the center. In that case, the strain would not be linear throughout the Jell-O, and would be much harder to calculate.

The disc of Jell-O will be loaded and the displacements measured. By placing a pan which has a larger radius that the Jell-O sample on top of the Jell-O, the total load will be evenly distributed over the entire area. One way to gradually add weight would be to slowly pour water or sand into the pan from a low height above the sample. The pan can be removed and weighed, and the deflections measured, at various points before failure. To be safer, entirely new samples could be used for each load value; then, if the sample is affected by repeated loading and unloading, it will not show up in the modulus calculations.

The deflection will be on a small scale, so a device capable of measuring small displacements is necessary. In order to minimize the error due to approximation, the device should be integrated into the Jell-O. There are two directions in which displacements must be measured: the change in diameter and the change in height. However, the measurement device must not exert any force on nor hinder the spread of the Jell-O.

To avoid missing minor deflections, the measurement device would ideally be depressed directly by the weight pan in the vertical direction, and expanded by the Jell-O in the lateral direction. In order for this to be effective, the tool must not cut into the Jell-O. One way to prevent this from happening is to put a lightweight thin sheet of a material much stiffer than Jell-O between the specimen and the measuring tool. On the top, the sheet should cover the entire surface (and therefore should not be forgotten when considering the load on the sample), so as not to apply an uneven load. On the sides, it should cover as little as possible, as the sheet should not create a barrier so large or heavy that the Jell-O spreads around it rather than push the measurement device outwards. The setup of the displacement measuring devices is shown in Figure 1.



(a) (b)

Figure 1: Measuring device set-up from the (a) top and (b) side of the sample

Using Hooke's Law, if the stress and strain on an object are known, the modulus of elasticity can be calculated. The stress on the disc of Jell-O is defined by the equation $\sigma = F / A$. The area, A, is found using the radius, and the force, F, is the weight of the pan full of sand or water. The strain on the disc is defined by the equation $\varepsilon = \delta / L_o$. The original length, L_o, is the original radius or height of the sample, as measured before placing any load (including the empty pan and sheet of stiff material) on top of the Jell-O. The displacement, δ, is the change in the radius or height of the mold as compared to the original sample. Thus, both stress and strain are known, and the modulus can be calculated using Hooke's Law, $\sigma = E\varepsilon$, or $E = \sigma / \varepsilon$.

By using a circular mold, the test will result in the ability to get multiple data points. If the load is applied uniformly, the Jell-O should spread out the same amount in all directions, resulting in a new circle of larger diameter. By taking multiple readings of the change in height and diameter of the same mold, an average value can be used that should have less error associated with it. Using multiple original radii and thicknesses will result in multiple stress and strain calculations that can be used to find the elastic modulus. These values should be equal, though as the thickness gets too great, the warping of the sides of the Jell-O will influence the results.

The amount of load on the Jell-O sample can be varied in order to determine whether the material is linear-elastic, and to what point. At some load, the Jell-O will fail rather than compress further, and it is at this point that it is no longer elastic.

This method will work to measure the modulus in compression. Unfortunately, it will not measure how the Jell-O behaves in tension. Measuring any material properties in tension is going to be difficult because of the weakness of the Jell-O, and the weight of the material. If a "beam" of Jell-O were to be created, it would most likely fall apart before any load was applied due to its own weight, as it is almost entirely made of water.

The liquid content of the Jell-O will also influence the modulus calculated. A very runny Jell-O will be much weaker than a very stiff (dry) Jell-O. Additionally, differing liquids will change the properties, as a liquid such as carbonated water would add air bubbles within the structure of the Jell-O. In order to use the results of this experiment in biomechanical fields, the type of Jell-O that is most similar to the tissue in question should be used.

On my honor as a student, I have neither given nor received unauthorized aid on this assignment.

Beth Abel Michelle Smith

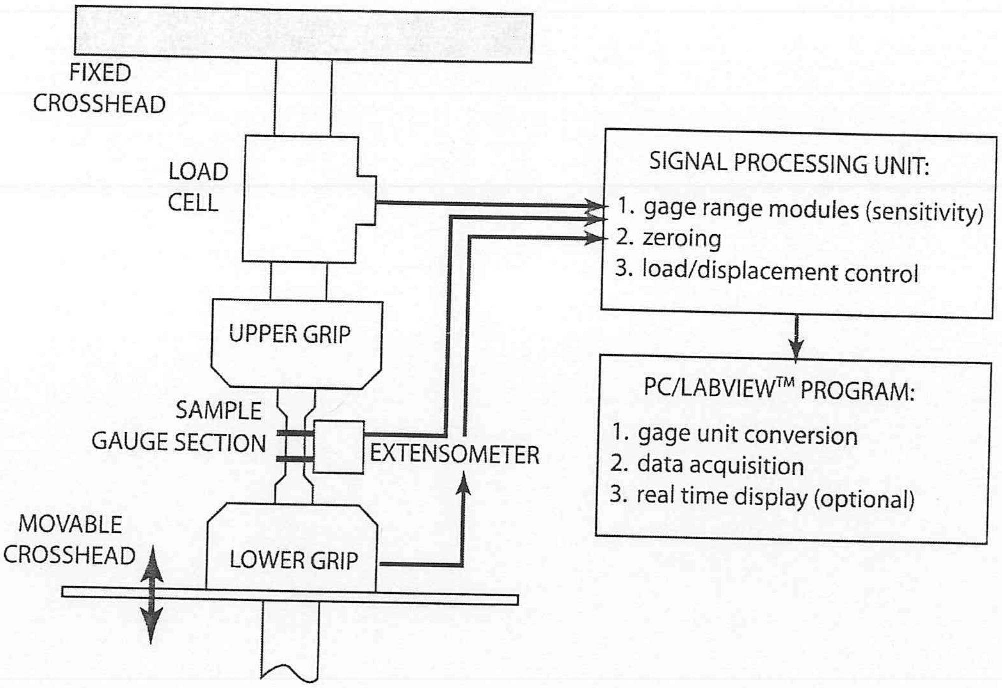
Beth Abel Catherine Hovell Michelle Smith

CE363 Materials Laboratory
Fall 2003

Experiment 2
Due: October 7th/9th

TENSION TESTING OF METALS AND POLYMERS

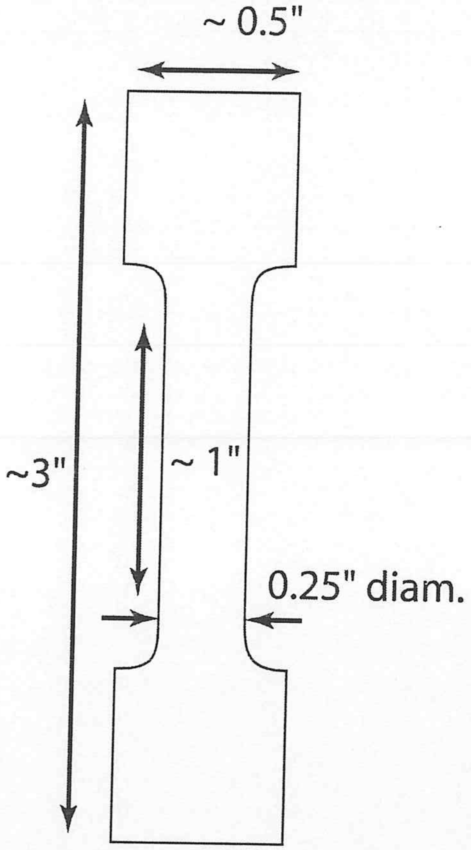
This laboratory experiment is an introduction to the most fundamental type of materials test, a uni-axial stress-strain measurement. A schematic of a typical experimental set-up is illustrated below.



The test is very simple: a specimen with a cylindrical cross section and flattened ends is gripped using hydraulic vises (labeled "upper grip" and "lower grip" in Figure 1). The *gauge section* refers to that part of the specimen that has uniform cross-section and can be used to relate to simply calculate the stress and strain from load and displacements, respectively. This lab uses a standard specimen geometry/size, described in the American Society of Testing and Materials (*ASTM*). The *nominal* dimensions of this specimen are shown on the next page. The sample is stretched by moving the lower grip, which is attached to a moveable *crosshead*. The experiment in lab will be *displacement-controlled*, since the lower grip is moved using the hydraulic driver in the test frame. Alternatively, load-control can be achieved by using closed-loop feedback control, as described under "Signal Conditioning and Processing" below.

Deformation measurement:

The deformation of the sample can be quantified a variety of ways. The simplest is to use *crosshead displacements*, which refers to the displacements of the lower grip, as monitored using the internal referencing system of the test frame. A more accurate approach is to fix an *extensometer* to the gauge section; this device outputs a voltage signal that is proportional to the displacements between two pieces (or "points") that are initially a precise distance apart. If an extensometer is used, the gauge section thus corresponds to the section between the two points.



Load measurement:

Load is measured using a load cell that produces an output voltage that is proportional to the force pulling on it (or pushing if the load cell is of the "*tension-compression*" variety, as it is here.) The maximum capacity of the load cell is 20,000 lbs. For some materials, this is larger than is required; the sensitivity of the cell can be adjusted by varying the proportionality constant between load and voltage output.

Signal Conditioning and Processing:

Displacements and loads are measured using *transducers* that convert mechanical quantities into voltages that are proportional to the mechanical stimulus (load or displacement). The *proportionality constants* between inputs (load,displacement) and outputs (voltage) are precisely determined by the manufacturers of the transducers; one must obviously know these to convert electronic data to physically meaningful quantities. In most cases, different electronic components are used to maximize the resolution of the measurement by ensuring that the inputs generate outputs in the 0-10 V range. For example, even though the maximum capacity of the load cell is 20,000 lbsf, one might wish to use a smaller *dynamic range* (e.g. for very compliant materials or small specimens.) In this case, the proportionality constant is reduced to improve the resolution of the load measurement: rather than using 0-10 V to cover 0-20,000 lbs, we can choose components such that 0-10 V corresponds to 0-2,000 lbs. The proportionality constants are adjusted in the LabView window that appears as Figure 3.

These factors are controlled using a signal processing unit that was manufactured specifically for our mechanical testing equipment. The signal processing unit allows users to set zero values, adjust the dynamic range, and in some cases, filter out electronic noise (such as that coming from fluorescent lights.) Alternatively, one can use commercial software/electronic packages that are integrated with a personal computer; the most widely used of these is called *LabView*. This software allows users to control voltages signals, establish feedback loops (e.g. to adjust displacement in real time – "on-the-fly" to maintain a specific load), and filter unwanted noise). In our case, we use LabView along with a *digital acquisition card* (DAC) in the PC merely to capture electronic data outputted from the signal processing unit. Thus, the transducers convert loads and displacements to electronic signals, the signal processor grabs them and dumps them to the PC.

Crosshead displacement → TRANSDUCER #1; Strain → TRANSDUCER #2

	LOAD CELL kips/volt	TRANSDUCER #1 in/volt	TRANSDUCER #2 unit/volt
set by gauge range (module)	☐	☐	☐
voltage at initialization (real time)	☐	☐	☐

Materials to be tested:

Three materials will be tested in the lab; an aluminum alloy, a steel alloy and a polymer. The precise composition of these materials can be determined by comparing our results with those in standard textbooks. The tests on the two metals samples is straightforward; the displacement is increased at a constant rate until the specimen fails; failure is defined in this case by the specimen breaking into two pieces.

Lab Write-Up:

1. From the data collected in class, and for each material:
    a. Plot two stress-strain curves (in the same figure): one calculating strains from crosshead displacement, and the other using the read-out from the extensometer.
    b. Determine the value of Young's modulus and compare with textbook values to identify each material.
    c. Determine the yield strength and yield point, using the 0.2% off-set method for steel and a 0.5% off-set for aluminum and polymer.
    d. Determine the ultimate tensile strength
    e. Determine the fracture strength
    f. Determine the percent elongation (maximum) and percent reduction in area.

strainrates:
-0.1hz
-1.0hz
-10hz
for polymers

2. For the polymer (and if things went well), determine the yield strength as a function of loading strain rate, and plot yield strength vs. strain rate on a log-log scale.
3. Discuss possible sources of error in the experiment and which of the above properties are most significantly affected, such as:
    a. Strain measurement (is crosshead displacement acceptable?)
    b. Deformation localization (i.e. where did it fail?)
    c. Dimensional tolerance (what if the dimensions are slightly off?)
    d. Alignment
4. Your lab write up should consist of:
    a. a brief abstract summarizing your results
    b. computer generated stress-strain plots with all axes and meaningful points labeled
    c. supporting calculations (e.g. how stress/strain are determined)
    d. a brief discussion of the possible sources of error (see #3) above.

CE323 Experiment #2: Tension Testing

DATA/RESULTS

Hovell

Completeness:

Stress strain plots 20 (20 pts)

Yield stress/yield strain 13 (13 pts)

Ultimate strength/fracture stress 12 (12 pts)

Yield stress vs. strain rate — (10 pts)

Percentage reduction — (5 pts)

Accuracy:

10 (10 pts)

Clarity & Labeling:

10 (5 pts)

Statistics:

— (5 pts)

DISCUSSION/PRESENTATION

Abstract: 5 (5 pts)

Error Discussion: 10 (10 pts)

Clarity of Writing: 5 (5 pts)

TOTAL: 85

CE 363 Materials Laboratory #2
Tension Testing of Metals and Polymers

Catherine Hovell
October 7, 2003

The modulus of elasticity of a material can be found by loading a sample of the material and measuring the reactions as the ~~beam~~ rod passes from elastic to plastic deformation, and finally fails. While the specimen is in the elastic deformation phase, a stress-strain plot will be linear, and the slope of the line is equal to Young's modulus for the material.

The stress-strain plot is made by measuring the displacements of a specimen loaded axially by a known load. The specimen should be held by the machine applying the load at sections that would have much larger failure strains than the part that is free standing. This helps ensure that the material will not fail underneath the grips, or in an area where the dimensions may not be fully known. Stress is defined as load divided by area, or $\sigma = P / A$. Strain is the change in length of a section of material divided by its initial length, or $\varepsilon = \delta / L_o$. In this situation, a tension-compression load cell is used to measure the load applied to the specimen, by converting changes in output voltages to physical values through the computer program LabView. This same program is used to analyze three different methods of measuring displacements so as to calculate strain.

Displacements are measured through three means: the movement of the crosshead, the voltage changes through an extensometer, and the distance between laser tags. In all cases, data points are sent to the experimenter through LabView, where slight calculations must be done to find strain. The extensometer, the most accurate, returns values already in the form of a ratio of displacement over original length, so to find strain, the value needs only to be multiplied by 100%. The crosshead displacement measurements send back values of how far the plate moves during the test run. Because there is an initial displacement, it must be taken into consideration in calculating strain, and the equation for strain becomes $\varepsilon = (L_x - L_1) / L_o * 100\%$, where L_x is the crosshead displacement at any load value x, L_1 is the very first value recorded, and L_o is the untouched length of the specimen. The laser tag measurements are very similar, as there is again some non-zero value associated with the initial distance between the tags. The difference in this situation is that $L_o = L_1$, as the length before loading is equal to the distance between the two laser tags, which need not be any value specific to the specimen, but rather, one specific to the lab setup and where the tags are initially placed on the specimen.

The repeatability of data with the extensometer is shown in Figure 1, a graph showing all six stress-strain curves made from extensometer measurements. Figure 2 shows all six curves again, but using data from the crosshead displacement. There is a great deal more variability, especially in considering the yield stress and strain. These two curves, as well as one produced with laser tag separation measurements, all from one steel test, are superimposed and shown in Figure 3. The laser tag strain measurements are very similar to the extensometer measurements, leading to a conclusion that those two measuring devices are more accurate than the crosshead displacement readings. Three similar plots are included as an Appendix for the aluminum tests. In those tests, there was less variability in the crosshead displacement readings, but the extensometer and crosshead values still do not match up. These inequalities are shown best when considering the failure strain, which varies greatly (up to 7%) between different measurement methods.

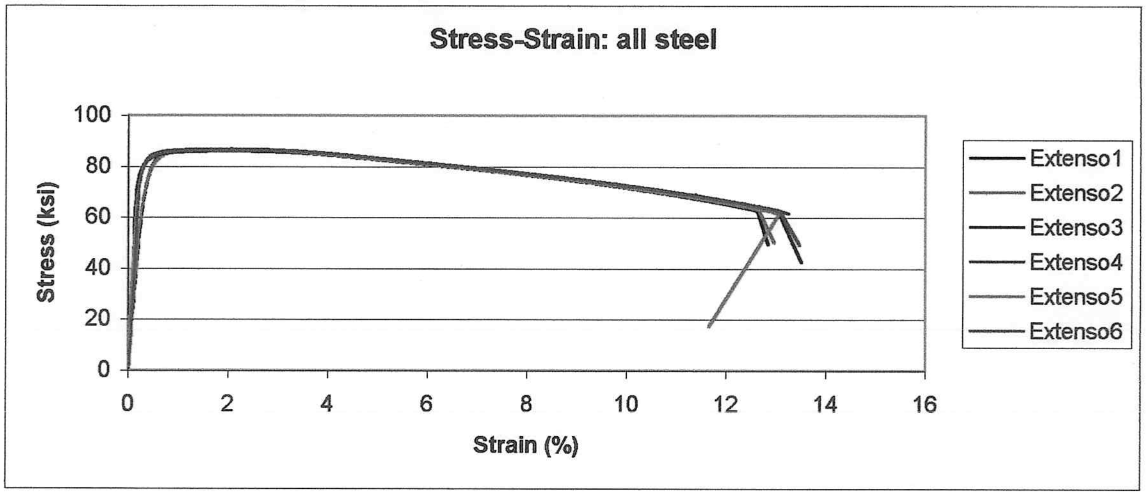


Figure 1: Stress-strain plot for all steel tests; strain measured by extensometer

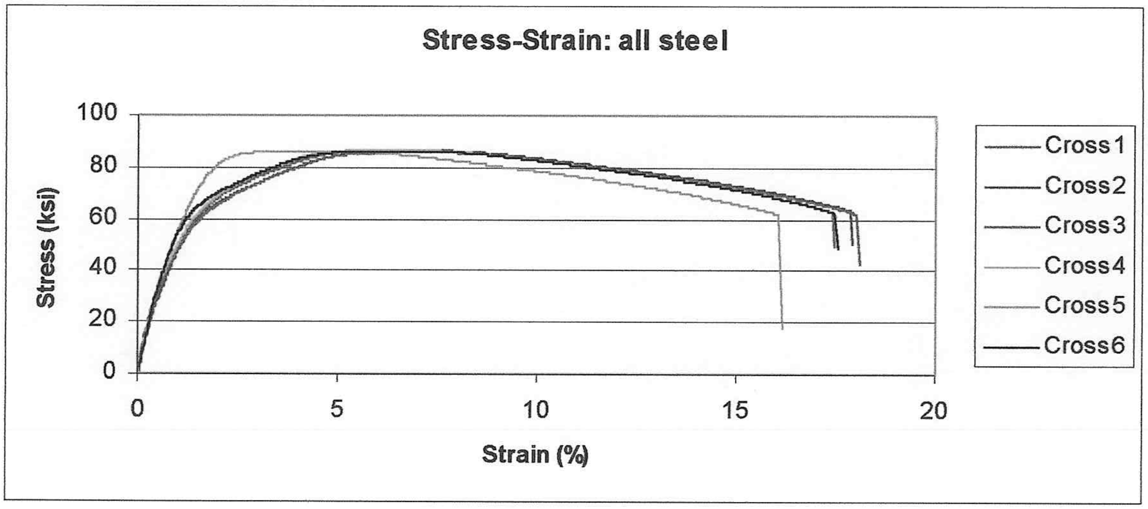


Figure 2: Stress-strain plot for all steel tests; strain measured by crosshead displacement

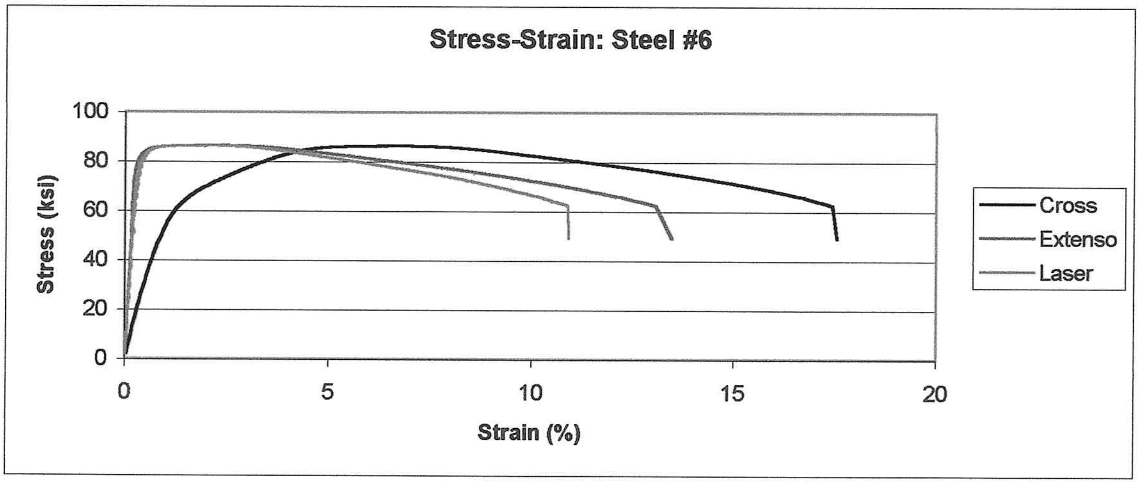


Figure 3: Stress-strain plot for sixth steel test, showing strain as measured by the extensometer, crosshead displacement measurements, and laser tag separations

Some key measurements, such as yield and failure strains, as well as an average value for measured modulus of elasticity, can be found in Table 1. These values can be compared to accepted values from a textbook; differences may be attributed to the fact that the composition of the specimen was not precisely known, and so the type of alloy may be incorrect. Ranges, or multiple values, are listed under textbook values in cases where more than one value is available. The failure strains varied for each measurement device. However, for both steel and aluminum, the extensometer had values between 10 and 15%, and the crosshead displacements measured strain to be between 15 and 20%. Although it is good to have multiple measurement devices set up, it is important to know which one is more reliable, as when they come back with significantly different results, the experimenter must know which values to rely on more heavily.

Table 1: Measured values, as compared to textbook values

Material	Modulus, ε		Yield strength, σ_Y		Ultimate strength, σ_U	
	Theoretical	Experiment	Theoretical	Experiment	Theoretical	Experiment
Aluminum	10.6×10^3 ksi	8×10^3 ksi	37-60 ksi	45 ksi	42-68 ksi	55 ksi
Steel	28×10^3 ksi	21×10^3 ksi	30-102 ksi	75 ksi	58-116 ksi	86 ksi

Error occurs in this experiment in two main areas: experimental setup and data collection. The specimen shape is designed so as to have maximum strain in the central area, rather than under the grips. However, if the ends are held too tightly, for instance, problems could arise where the cross-sectional area of the specimen under the grip is smaller than the free-standing area, and the material could fail where displacements could not be accurately measured. Fortunately, it does not appear that this happened in any of the tests analyzed here. Another concern in the initial setup is the orientation of the specimen – it is important that it is as vertical as is possible. If the specimen was at an angle, but the load was applied directly on the y-axis, new stresses and strains would be introduced and the distributions would not be uniform. Lastly, it is important to check and calibrate according to initial conditions. The machinery used has the ability to load the sample while merely gripping each end, applying strains that are not considered in analyzing the data, but affect the sample.

Errors due to data collection were mentioned briefly before, and depend most on the three different methods employed in this test to collect strain data. For one thing, the larger the range of a variable, the less accurately the data acquisition devices will be able to measure the data. Each device has a 10 V range of input voltages, which must be spread over the entire range, regardless of its size. Therefore, this acts as a limiting factor for how accurate the data can be. As well, the crosshead displacement measurements add another factor to the strain calculations; fortunately, it is one that could potentially be minimized in the experimental setup. If the ends of the specimen are not contained within the grips at either end, they can potentially stretch as well. Since the ends are not of the same cross-sectional area as the gauge section, this changes how stresses are distributed through the specimen. As well, the unstretched length of the specimen becomes a value other than the length of the gauge section, changing the strain measurements and calculations. The variability of data using three different data gathering methods show how the error due to experimental setup (among other things) can be minimized by choosing the right method of recording data points. The most crucial step in this experiment is therefore not running the actual test, but deciding how to set it up so as to get the most accurate, useful, and repeatable results.

On my honor as a student, I have neither given nor received aid on this assignment.

Appendix: Figures for aluminum and polymer samples

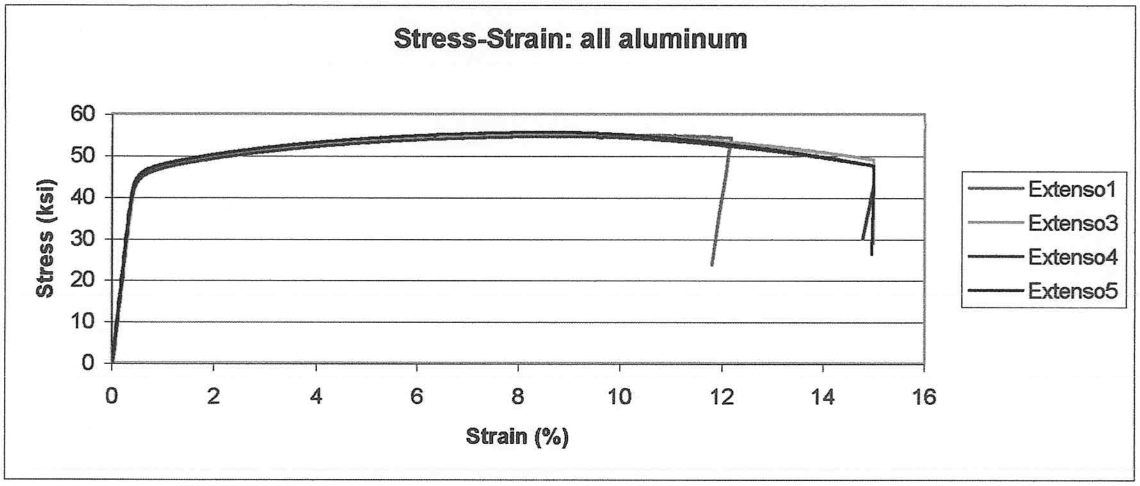


Figure 4: Stress-strain plot for all aluminum tests; strain measured by extensometer

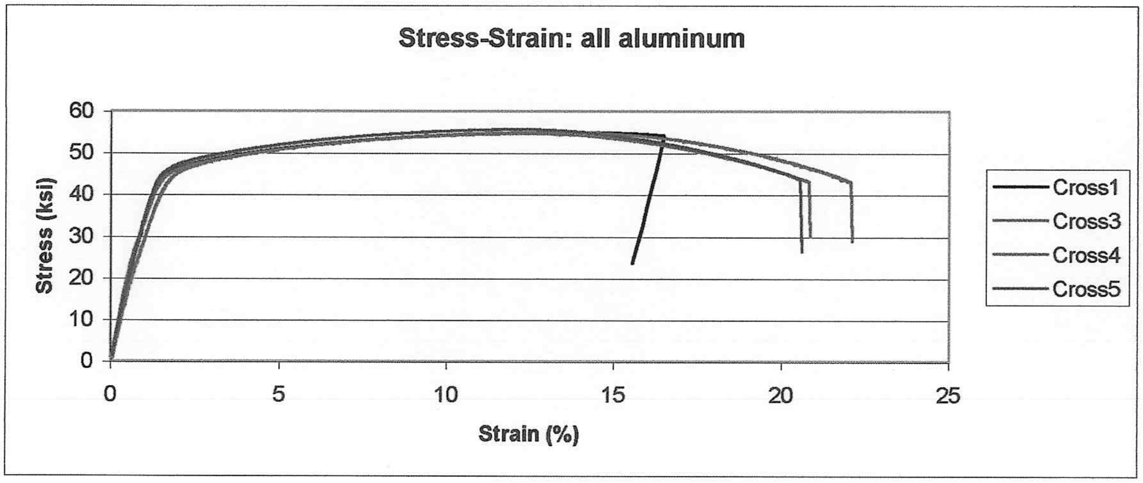


Figure 5: Stress-strain plot for all aluminum tests; strain measured by crosshead displacement

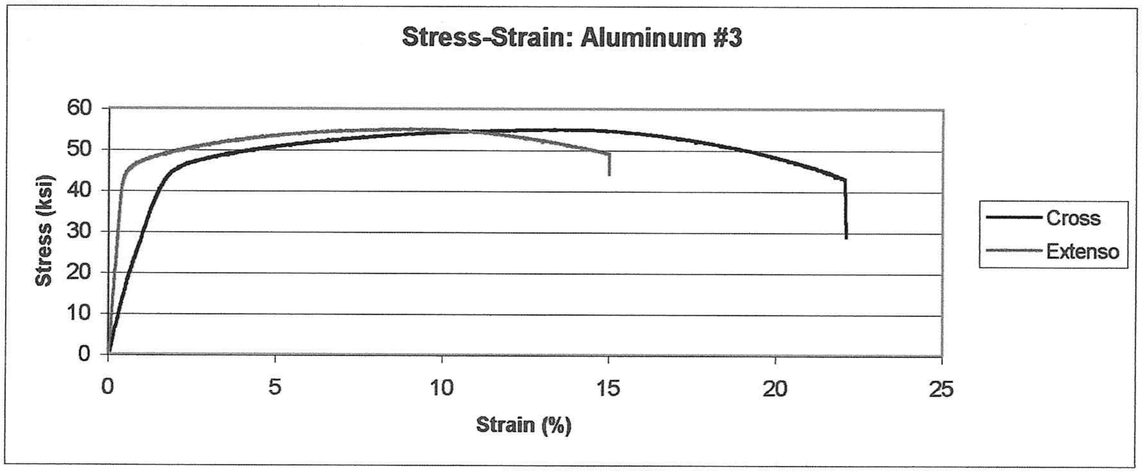


Figure 6: Stress-strain plot for third aluminum test, showing strain as measured by the extensometer, crosshead displacement measurements, and laser tag separations

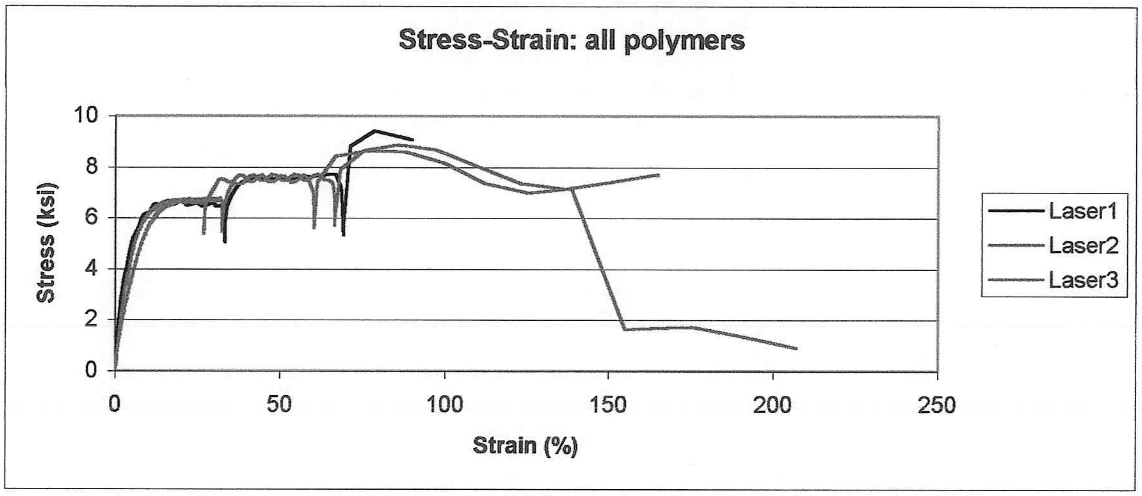


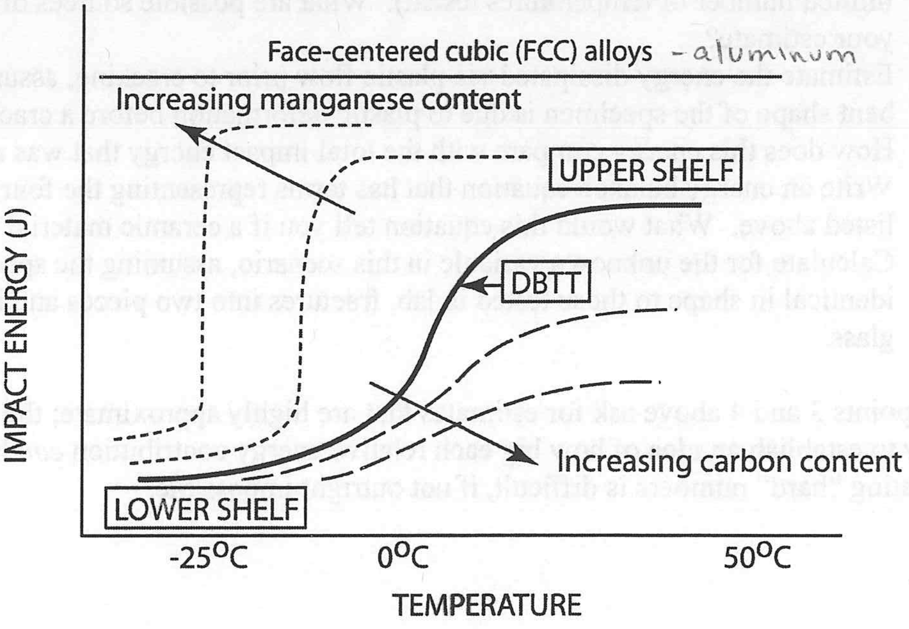
Figure 7: Stress-strain plot for sixth steel test, showing strain as measured by laser tag separations at strain rates of 0.1, 1.0, and 10 Hz

CE363 Materials Laboratory **Experiment 3**
Fall 2003 **Due: October 21th/23th**

CHARPY IMPACT TESTING OF METALS AND POLYMERS

This laboratory experiment is an introduction to the simplest way to characterize the fracture energy (or impact energy) of specimens. A large, heavy pendulum has a wedge shaped hammer in the middle. The pendulum is raised to a measured height and let loose; after impact, the maximum height is noted. The energy absorbed during fracture is associated with the change in potential energy of the hammer. The test provides an excellent mean to do two things: (*i*) compare two specimens (usually, but not always, two different materials) and (*ii*) changes in fracture energy with temperature. In this lab, you will do both. Charpy impact tests will be run on: a steel alloy, an aluminum alloy, and a polymer. Testing will be done at three different temperatures: room temperature, cold specimens cooled with liquid nitrogen, and heated specimens.

The goal is to determine the ductile-to-brittle-transition-temperature (DBTT). This transition temperature is exactly what it sounds like: the critical temperature where the fracture changes from brittle failure to ductile failure. Each type of failure has a characteristic appearance, as will be discussed in class. The DBTT is illustrated in the schematic below; it varies greatly with the crystal structure and alloying elements added to the base material (iron or aluminum).



Note that the Charpy impact test is ***not*** a valid means to determine the *fracture toughness* of the material, which governs fracture initiation and subsequent crack growth. (The *initiation fracture toughness* is the energy per unit area to start the crack growing; the *work of fracture* is the energy per unit area required to keep the crack growing.) Why? Because there are a number of 'events' that occur following impact that each dissipate

Yay Materials Lab

7 October 2003

notch not made professionally

energy losses: sound
heat
kinetic

crappy alloy
lots o' stuff in it

modulus is temp
dependent

	Aluminum:		Steel:		Polymer:	
ROOM	16 ft.lb	cup + socket	26 ft.lb	45°	7.25	– no break straight ↓
	16,25 ft.lb		27,75 ft.lb		9,5	
	18 ft.lb		26 ft.lb		9.25	
COLD −50°C ish	162	– no break plastic yielding (in backwards)	8	slow	1.25	broke! really straight fracture
	20.75		4	get caught up, more at angle	1.75	
	20		8	middle jaggedness	2.25	only one side move
HOT 100°C	19.75		22	straighter fracture surface	7	lots of kinetic energy - fast!
	19		20		7	(no breaks)
	17		22			

yield → fracture initiation → work of fracture → kinetic energy

CE363 Materials Laboratory **Experiment 4**
Fall 2003 **Due: TBA**

Extra credit

FRACTURE TOUGHNESS TESTING "CON QUESO"

This laboratory experiment (conducted in your very own home) is an introduction to the simplest way to characterize the fracture toughness of brittle specimens. A double cantilever beam fracture specimen is illustrated below. The energy release rate is defined as:

$$G = -\frac{1}{b}\frac{dU}{da} = -\frac{1}{b}\frac{d}{da}\left(S.E. + W_{ext}\right) \qquad (1)$$

where $S.E.$ is the strain energy in the beam and W_{ext} is the external work done by the loading mechanism.

1. Using elementary beam theory, show that the strain energy stored in one of the tip-loaded cantilever beams is equal to:

$$S.E. = \frac{2P^2a^3}{Ebh^3} \qquad (2)$$

Also, show that the deflection of the load-points (where P is applied) is equal to:

$$\Delta = \frac{4Pa^3}{Ebh^3} \qquad (3)$$

2. Using the expressions above, show that the following expressions for energy release rate can be obtained:

$$\text{Load control: } G = \frac{6P^2a^2}{Eb^2h^3} \qquad (4a)$$

$$\text{Displacement control: } G = \frac{9E\Delta^2h^3}{2a^4} \qquad (4b)$$

3. Determine the fracture toughness (denoted as G_c) of ordinary cheese (of your choosing) by measuring the loading parameters needed to initiate crack growth (and setting $G = G_c$) using either of the results above. You will need to figure out how to: (*i*) create an appropriate specimen that includes an initial crack, (*ii*) impose and quantify your desired loading parameters, and (*iii*) observe the relationship between load and crack length (e.g. identify critical loading parameters that initiate crack growth).

4. Provide a brief discussion of your approach, noting its successes and limitations.

CE363 Materials Laboratory **Experiment 5**
Fall 2003 **Due: Friday, 12/5**

COMPRESSION STRENGTH OF WOOD

Part One:

Provide a review of the technical concepts and data in Gordon's chapter on wood (Ch. 6) – no longer than three pages. Your write-up should summarize:

- The role of wood as a structural material in the U.S. and elsewhere
- A brief description of the microstructure in wood and the role that each component plays in carrying load
- A summary of the deformation mechanisms that lead to failure in wood
- A brief description of the role of water and processing (e.g. drying, treatment, etc.) on the compressive strength of wood.

The goal of this part of the lab is to summarize the behavior of wood, both qualitatively and whenever possible, quantitatively. Towards this end, a bulleted list of the data and trends that Gordon reviews would be most helpful.

Part Two:

Summarize the data collected on dry and wet wood, transverse and parallel to the grain. Your summary should include stress-strain curves, with modulus and failure stress noted on the plots. Compare your data with published values for Douglas fir. Briefly (1/2 page) summarize the relative accuracy of the measurement and its potential impact (or lack thereof) on choosing a failure stress on which to base designs.

Materials Lab

11/4/03 3:30-5

wet is much
more elastic

dry wood - 17% by weight H_2O vs. wet wood - 37% H_2O

$l_0 = 6$ in $A = 2" \times 2" \times 8"$

H_2O does not
matter for strength

mode I - parallel to the grain (failure)

load (kip) disp. R L (in)

2×2×2 ① dry, perp to grain ② wet, ⊥ to grain

0	0.0145	0.0035	0.00~~65~~75	0.003
1	0.037	0.0275	0.0295	0.040
2	0.042	0.031	0.0625	0.0305
3	0.047	0.038	0.137	0.0995
4	0.068	0.056	0.295	0.246
5	~~0.013~~ 0.13	0.12		

×2×8 ③ dry ∥ to grain ④ wet, ∥ to grain

0	5×10^{-4}	1	0	~ not such a hot pt. $E \sim \infty$
5	10×10^{-4}		0.001	
10	20×10^{-4}		0.0025	
15	30×10^{-4}	16	0.0040	
20	4×10^{-3}	22	0.0060	
25	5×10^{-3}	26	0.0070	
30	65×10^{-4}		0.0080	
35	7.5×10^{-3}		0.010	— buckle load
40	9×10^{-3}			
45	10.5×10^{-3}			
52	.014			

CE363 Materials Laboratory **Experiment 6**
Fall 2003 **Due: Friday, Dec. 12th**

COMPRESSION STRENGTH OF CONCRETE

This laboratory will quantify the compressive response of concrete with different contents of water, cement, aggregate and sand. The following concrete mixes will be tested in lab:

	Samples	Water, gals	Cement, lbs	Aggregate, lbs	Sand, lbs	
Mix #1	1a-1f	0.22	5.16	9.95	5.67	6ksi
Mix #2	2a-2f	0.22	3.61	9.95	6.97	4ksi
Mix #3	3a-3d	0.22	2.58	9.95	7.84	2.1ksi
Mix #3n	3N1, 3N2	0.22	2.58	0.	7.84	2.1 ksi

Each class will test two samples of each mix, except for Mix #3; two of these six samples were cast without aggregate. You should acquire the compressive strengths of samples tested in other sections: be friendly and ask nicely.

1. Provide compact and detailed plots of the stress-strain response of the different mixes, with some indication of variability (e.g. average stress-strain response with experimental range superimposed.)

2. Solve for the volume of concrete that was mixed, assuming an aggregate size of 0.75"

3. Estimate the strength of the different mixes, assuming that the desired slump of 2" was achieved, and that the mix/casting process introduced entrained air into the mix.

4. Provide a **brief** discussions of your results, including the following:

    a. A description of stress-strain behavior, noting any differences in qualitative trends for the different mixes
    b. A comparison of theoretical and experimental compressive strengths
    c. Reasons for discrepancy between theoretical strength and compressive strength, including a quantitative assessment of possible mixing errors.
    d. A brief description of failure sequence and post-yield response of the samples.

CE 363 : Concrete

4.5 ksi = 18 kip
3.0 ksi = 12 kip
2.0 ksi = 8 kip

Mix #1 L E

1.1495 in = L_o

A. fail ~ 14 kip ~~[illegible]~~

$\Delta L = 0.07$ in

Mix #1

$L_o = 1.1190$ in

B. 1.1170 in 11 kip

13.5 kip fail

Mix #2

$L_o = 1.1108''$

A. 1.1104" 4 kip

(1.0780) 5 kip fail

Mix #2

B. $L_o = 1.1303''$

1.1312 2.5 kip fail

Mix #3 w/ aggregate

A. $L_o = 1.1265$ 2.5 kip

~~[illegible]~~

B. $L_o = 1.1216$ 3.3 kip

3N1 (no agg.) L_o 1.1390 3 kip

$0.22 \text{ gal } H_2O \times \frac{1 \text{ ft}^3}{7.48 \text{ gal}} = 0.029 \text{ ft}^3$

$5.16 \text{ lb cement} \times \frac{1 \text{ ft}^3}{190 \text{ lb}} = 0.027 \text{ ft}^3$

$9.95 \text{ lb agg} \times \frac{1 \text{ ft}^3}{170 \text{ lb}} = 0.059 \text{ ft}^3$

$5.67 \text{ lb sand} \times \frac{1 \text{ ft}^3}{160 \text{ lb}} = 0.035 \text{ ft}^3$

0.096 ft^3